Credit spread shocks: how big and how often?

The second half of 2007 saw violent moves in credit spreads. In the fallout, there has been much discussion about how to estimate the probabilities of these severe events, but few conclusions have been obtained beyond the fact that historical data is often unhelpful. In this article, Richard Martin argues that the credit default swap curve prices much of this information in, and that these probabilities can be extracted using a Lévy-based structural model without recourse to historical data; by contrast, reduced-form approaches cannot do this. He also introduces the concept of 'market-implied value-at-risk'.

A central question in managing the mark-to-market risk of a credit trading book is ascribing probabilities to big spread moves or, equivalently, finding mark-to-market value-at-risk at high confidence levels. It is worth winding the clock back to early July 2007 (fortunately one has such luxuries when writing articles) to get an idea of what different models say, or said, about such events.

Suppose, for added realism, that you are the quant responsible for managing desk risk, and the head trader wanders by your desk: “We’ve been talking about our financials exposures, which are pretty big”, he says, “Can you put a number on how much we might lose, at say 99% or 99.9% confidence over the next few months, through spread widening?” Over to you, and the data at your disposal is figure 1, the recent spread history for one particular issuer (a monoline insurer).

Clearly, you have quite a problem on your hands, because the trader is worried not about a basis point or two, but several hundred basis points, and nothing like that has happened before. Nevertheless, you start by proceeding down the well-trodden path of ‘historical VAR’, and fit a lognormal hazard rate model to the available data:

\[ d\lambda_t = k (\theta(r) - \lambda_t) dt + \sigma\lambda dW_t \]  

(1)

It is a close enough approximation to say that a lognormal hazard rate model gives five-year spreads that are lognormally distributed, with the same volatility, so you can infer the volatility pretty much directly from the data by taking an exponentially weighted moving average of square relative spread changes. When you do this, the volatility comes out at about 90%, which is the important parameter for explaining short-term moves. From the current level of 80bp, the probability of a one-month move out to (say) 400bp is then easily obtained as the probability of a \[ \ln(400/80)/(0.9/\sqrt{12}) = 6.2 \] standard deviation move by a Gaussian variable, which occurs with probability about 0.0000000003. Well, clearly you cannot go and say something like that. The rolling VAR estimates are also shown in figure 1 at two confidence levels, and they seem unreasonably optimistic. In fact, this issuer went to 950bp in early August 2007.

Accordingly, it seems reasonable to consider a model with jumps, to get away from the Gaussian picture. The problem with this is that if no big jumps have ever occurred, a backward-looking model is impossible to calibrate from history and one is in effect guessing the jump intensities, even if the guess is dressed up in a model. Now the model (1) is a ‘converted’ interest rate model with the hazard rate \( \lambda_t \) substituted for the short interest rate \( r_t \), but credit spreads are not interest rates. Maybe by modelling the default event, and hence the physical origin of credit risk, some other insight might be gleaned?

This brings us to the structural models, which typically model the default event by the value of the firm hitting a barrier. The credit spread is thereby viewed as the price of a barrier put option on the firm’s assets. Spread and spread volatility must be connected in structural models, because the more volatile the assets of the firm, the higher the credit spread, and because the credit spread is a function of the firm’s asset level, the more volatile the credit spread will be. Note that this explains very well why credit volatilities vary so differently between regimes: when spreads are tight the embedded put option is way out-of-the-money and has very little price volatility, but when spreads are wider it is closer to being in-the-money and then its price volatility is very much higher. So much for volatility, but we are bothered about the bigger spread moves.

A key issue with structural models is how to make the term structure of default in the model match that seen in the market. It is well known that a recipe that does not work is the use of Gaussian firm-value dynamics with a flat barrier, as in that case the probability of immediate default is zero and the credit default swap (CDS) curve is too tight at the short end, contrary to what is observed in the market. How best should this be corrected? We wish to use a time-homogeneous model, that is, one that as far as possible does not have maturity-dependent parameters, as such
variation would have questionable economic interpretation (see figure 2). For example, making the short-term volatility infinite, and allowing it to decay over time, would fix the problem of the short-term spread, but does not make much sense (why is the world so much more volatile today than it is predicted to be in a week’s time?). Similarly, one could play with the shape of the barrier to make the model fit, so that the firm has great liabilities today but that these are predicted to decay over time. This also makes little sense. Finally, one might seek to make the barrier volatile, with the interpretation that the true asset to liability ratio of the firm is unknown, and that the firm might be close to default (Finger et al, 2002), but to make this work properly one needs to introduce dynamics for the barrier, and again the interpretation is dubious.

Uncertainty in the position of the barrier was a fashionable device for explaining credit spread back in 2002/03 when there was widespread uncertainty about the reliability of information on the balance sheet, as exemplified by the defaults of Enron, WorldCom and Parmalat. The last of these is considered by Brigo & Morini (2006) along with a high-grade telecom issuer, Vodafone (CDS = 28/43bp at two-year/five-year at that time). To fit Vodafone required the leverage to be 38% five-sixths of the time and 87% for the remaining one-sixth. Dozens of similar companies had this CDS level in 2006. So either one-sixth of the high-grade corporate universe had almost 90% leverage – in which case they would presumably have defaulted by now (which they have not) – or the market was at that time pricing in a one-sixth chance of the whole universe having such leverage, which systemic risk would presumably have been reflected in the super-senior spread (which at 2–5bp it was not). Either way the model makes little sense except in isolated examples.

The most promising route seems to be to incorporate jumps into the model, thereby changing the process followed by the firm value from a geometric Brownian motion to a geometric Lévy process. One thing that is apparent is that the shape of the curve plays an important role: the flatter the CDS curve (higher short-term spread), the higher the jump intensity needs to be made in order to fit the curve. This is in total contrast to (1), where the shape of the curve is parameterised through θ(t) and has nothing to do with the spread dynamics. Also, in the structural model, assigning probabilities to large spread movements is an interpolation, not an extrapolation. This is seen from figure 3, where the probability mass in the ‘black part’ is known. In hazard-rate modelling one conditions on survival, thereby splitting off the ‘black part’ and losing any conception of what the spread distribution might be.

The use of Lévy processes in structural models is not new, but previous literature has only concentrated on fitting the CDS curve and/or equity option markets. The application to mark-to-market VAR, where it would probably be most beneficial, has not so far been considered, except in a publication by Credit Suisse in 2007. This leads to the interesting avenue of a ‘market-implied’ or ‘model-implied’ VAR as distinct from the historical methods usually employed (as the model is calibrated to the market in this case, we use the terms ‘market-implied’ and ‘model-implied’ interchangeably). We shall also discuss how to implement this kind of VAR.

The reader is referred to Schoutens (2003) for an overview of Lévy processes, and to Kou & Wang (2003) and Carr et al (2003) for details of the double-exponential model and the Carr-Geman-Madan-Yor (CGMY) model respectively. A recent paper by
Madan & Schoutens (2008) uses the same prescription (CMY) that the present author has been using for some time and will use here, though with different numerical methods. As a general introduction to structural models and their application in credit trading, the reader is referred to Credit Suisse’s publication on CUSP² (Martin, 2007).

**Structural model: parameterisation and calibration**

**Basics.** We model the firm value as a geometric Lévy process:

\[ A_{t+dt} = A_t \exp(dX_t) \]

where \( dX_t \) denotes the fractional change in firm value between times \( t \) and \( t + dt \). The increments \( dX_t \) are independent for non-overlapping periods. The distribution of \( dX_t \) is not normal, but we shall come to that presently.

The initial firm value \( A_0 \) is equal to the sum of the market capitalisation of the firm and the present value of all its debt, which is assumed to be perpetual and of total face value \( F \). For these purposes it is the leverage, that is, the ratio \( \ell = F/A_0 \), that is important. We assume that \( \ell \) is known. If the objective were to estimate the default probability of the issuer from its capital structure and volatility of assets, we would find uncertainty in leverage to be an awkward problem. However, we are calibrating to the market default probability of the issuer from its capital structure and replace the volatility term with the following expression:

\[ \nu(dx) = C\gamma^{-1}e^{-\gamma dx}, \]

where \( \gamma < 1 \) is called the shape parameter, and the generator is \( L(u) = \mu u + \alpha^2(1 - (1 + \beta u)); \) further details are given in Madan & Schoutens (2008). This family includes the exponential jump-model, and the gamma and inverse Gaussian models \( \gamma = -1, \frac{1}{2} \) respectively. In the limit \( \mu \to +\infty, \alpha \to +\infty, \beta \to 0, \) with \( \mu = \alpha \beta \) and \( \alpha \beta^2 \) fixed, we end up with a Brownian motion again, as \( L(u) \) reduces to a quadratic in that limit. So the Brownian motion is still present in the resulting family of processes.

**Parameterisation.** There are four parameters to be specified, and we argue that \( \gamma \) is of secondary importance and can be fixed, and that \( \mu \) is set by risk-neutrality arguments. This just leaves two \( (\alpha, \beta) \), which are calibrated from two points on the CDS curve. This gives reasonable flexibility without too many parameters.

To set the drift, we assume that the firm pays a continuous coupon \( \varphi \) to the debt-holders. If the face value of the debt is \( F \), the spot-forward parity relation for the firm is approximately:

\[ E_t[A_{t+dt}] = A_t (1 + r - \varphi) dt, \]

\[ r^* = \frac{\varphi}{1 - \varphi} \]

This imposes the condition \( L(1) = r - r^* \), and now \( \mu \) is set.

Now it is convenient to reparameterise a bit, because \( \alpha, \beta \) have no obvious financial interpretation, so we proceed as follows. The second cumulant of the Lévy generator, that is, \( L''(0) \), has an interpretation as a squared volatility, and is exactly that in the Brownian case. In the CMY model, this is \( L''(0) = (1 - \gamma\alpha^2) \), which we denote \( \sigma^2 \).

The third cumulant \( L'''(0) \) is, on division by the \( \frac{1}{2} \)-power of the second, interpretable as a skewness⁴, which we denote \( \tilde{\sigma} \tilde{\gamma} \).

We can therefore express everything in terms of \( \gamma \) (fixed), \( \alpha, \beta \).

**Firm-value dynamics.** For the usual Brownian motion with drift \( \mu \) and volatility \( \sigma \), which is a special case, \( dX_t \) is normally distributed with mean \( \mu dt \) and variance \( \sigma^2 dt \). For more general Lévy processes, the distribution is most easily specified through the generating function \( L(u) \) given by \( logE[\exp(\mu u dX_t)] \), where \( u \) is a complex variable. In the case of the Brownian motion, \( L(u) = \mu u + \frac{1}{2}\sigma^2 u^2 \). We wish to incorporate downward jumps into the model, and replace the volatility term with the following expression:

\[ L(u) = \mu u + \int_{\mathbb{R}} (e^{\nu t} - 1) \nu(dx) \]

in which \( \nu(dx) \) is called the Lévy measure and can be thought of as the intensity of jumps of size \( x \), all sizes \( x < 0 \) being possible. We have been using the CMY model, in which the Lévy measure is given by

\[ \nu(dx) = C\gamma^{-1}e^{-\gamma dx}, \]

where \( \gamma < 1 \) is called the shape parameter, and the generator is \( L(u) = \mu u + \alpha^2(1 - (1 + \beta u)); \) further details are given in Madan & Schoutens (2008). This family includes the exponential jump-model, and the gamma and inverse Gaussian models \( \gamma = -1, \frac{1}{2} \) respectively. In the limit \( \mu \to +\infty, \alpha \to +\infty, \beta \to 0, \) with \( \mu = \alpha \beta \) and \( \alpha \beta^2 \) fixed, we end up with a Brownian motion again, as \( L(u) \) reduces to a quadratic in that limit. So the Brownian motion is still present in the resulting family of processes.

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**Survival probability and CDSs.** To calculate CDS spreads and the like, we must calculate the probability of survival to date \( T \), which is given by:

\[ Q(T) = P \left[ \min_{t \leq T} A_t > A^* \right] \]

In general, this is difficult to calculate in closed form when \( A_t \) follows a Lévy process (see, for example, Sato, 1999, for a discussion of

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² CUSP stands for Credit Underlying Securities Pricing and is a trademark of Credit Suisse

⁴ Not exactly, because the coupon payment is not in reality proportional to the firm value; however, it works to be in the model premisse. This is a minor issue.

Note that \( \gamma > 0 \) here, but because the jumps are downward, the skewness of the asset process is \( -\gamma \) (and hence negative). This should not cause confusion.
the Wiener-Hopf method) except for a few special cases, the exponential jumps ($\gamma = -1$) being one. Experience suggests that the problem is best tackled numerically, on a grid, in which both axes (time and log firm value) are discretised. The distribution of firm value at the next time step is obtained from the current one as follows. First, convolve the current distribution with the distribution of the incremental asset process $dX$. This is best done in Fourier space, as the Lévy generator is known in closed form and the fast Fourier transform algorithm makes the computation very fast. Second, deal with default by eliminating any probability mass below the barrier.5 One thereby obtains the solution of the forward Chapman-Kolmogorov equation for the evolution of the probability density, or Arrow-Debreu prices, starting from a given initial condition. The forward Chapman-Kolmogorov equation is used for working out the survival probability and hence the CDS.6 The backward Chapman-Kolmogorov equation, for the expectation of some function conditional on many starting points simultaneously, is solved the same way as a two-step procedure (only in backwards time). It is used for finding the distribution of present value of a contract at some future horizon, working backwards from its expiry. Both are needed here.7

Examples and model sensitivities

\textbf{Performance of ‘model VAR’}. Let us now apply all this to the monoline example, first looking at July 2007 to examine the VAR estimates. The model parameters needed to fit the CDS curve (2Y = 360bp, 5Y = 800bp) are, with $\gamma = 0$: $\ell = 0.30$, $\delta^2 = 0.45$, $\kappa = 2.0$. Figure 4 shows the distribution of present value of a five-year short-CDS contract at a one-month horizon. Notice first the extreme asymmetry: the maximum upside is a handful of basis points from spread tightening and the maximum downside is default with zero recovery. A move out to 400bp in one month, causing a loss of ~16% of notional, is not vanishingly small, being accorded a probability of ~0.2% (for three months, it is ~0.5%). Figure 5 shows the one-month horizon 99% and 99.9% VAR estimates on each day, going back over the previous years, and they are consistently higher than in figure 1. (Note the different vertical scales.)

We can also obtain the model-implied spread transition matrix (in effect, the reduced-form model contained inside the structural model). It is easy enough to calculate the transition matrix for the firm value, row by row for different starting points. There is a one-to-one correspondence between the five-year CDS spread and firm value, thereby making the five-year CDS spread the measure of credit quality6 (see table A). Exact comparison with rating transition matrices is awkward as they are saying different things, but as an example let us compare the model probability of a one-year transition from a five-year spread of 30bp to 700bp with the historical rating transition probability from A2 to B2. The model probability is about 0.4%, and the historical one is about 0.045% (Moody's Investors Service, 2007, exhibit 15), which is 10 times smaller. In other words, the model suggests that extreme moves in spread are much more likely than big downgrades. This is probably well appreciated, but hard to quantify.

The conclusion is that a structural model with jumps ‘prices in’ big jumps when it is fitted to the CDS curve, and can be made to do so without reference to historical data. If these are used in place of empirical distributions to determine a “model VAR”, on what does this VAR depend?8 The evidence and conclusions are set out below.

\textbf{Curve shape is important. Flatter curves indicate more jump risk.} If we increase the two-year spread from 36bp to 60bp, the five-year spread remaining fixed at 80bp, and recalibrate, the intensity of very big jumps has to become higher, which is reflected in a higher skew coefficient $\kappa (\ell = 0.3, \delta^2 = 0.43, \kappa = 4.0)$. Note that the intensity of smaller jumps must reduce, as the expected loss is the same (because the five-year spread is being held fixed); to make all adverse mark-to-market moves more likely would require a higher five-year par spread. To repeat what we said before: this deduction is not valid for hazard-rate models, as in those, curve shape and curve dynamics have nothing to do with each other (because the hazard-rate drift controls the curve shape and the stochastic term controls the dynamics).

\textbf{Assumed leverage is less important.} We are already calibrating to the market spread, which prices in a certain leverage (or equivalently a certain volatility of firm value). Moving the assumed leverage up and down simply causes the calibrated firm volatility to move oppositely. Figure 6 demonstrates that the effect of assumed leverage ($\ell = 0.2, 0.4$ is small, provided the model remains calibrated to the same CDS curve (2Y, 5Y = 36, 800bp as before).

\textbf{Lévy measure is fairly unimportant.} Obviously the choice of Lévy measure must affect the conclusions. However, within the family considered here there is very little variation in the probabilities

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6 What has just been described is sufficient for a CDS contract with fixed recovery. In the implementation used here, the recovery is dependent on the firm value just after the barrier has been crossed, and the process of jumps makes the distribution of recovery non-trivial. The problem can still be solved using the method described provided that at each time step one records the probabilities at all grid points below the barrier, before averaging them. The default log PV can easily be calculated then.

7 Vanilla options can be priced using either the forward or backward procedure; we need $E_d [\tau X(t) | X(0)]$, so either iterate $\tau$ forward (forward equation) or $\tau$ backward (backward equation).

8 To assess the performance of a five-year CDS contract, for different statements in firm value, the columns of the transition matrix would instead be identified with four-year CDS spreads, thereby taking the roll-down into account.
Towards model-implied VAR

The market appears to price jump risk into the CDS curve. One can therefore ascribe a reasonably sensitive probability to a big spread jump even though it has never happened before, and clearly the numbers produced are more sensible than those of a lognormal hazard rate model. This suggests the use of such models in credit VAR, and indeed experience in the last two years has shown that this approach works well, particularly if equity option-implied volatilities are also incorporated into the model.

Although this might seem like a departure from ‘standard VAR’, it is not. The purpose of VAR models is to ascribe probabilities to future events, and then evaluate the risk of the current set of trading positions in the light of these. If models calibrated to the market provide better estimates than historical data – potentially the case in a situation where no such bad events have previously happened – then it is common sense that these should be integrated into the model. As such, what is going on in the equity and equity options markets is also useful: it is intuitively sensible that if equity volatilities suddenly spike up then future credit spreads and spread volatilities are probably too high, and indeed the evidence for this is strong, but this will be the subject of a future publication.

A potential criticism is that, because the jump probabilities are obtained by calibrating to market instruments, they contain a risk premium and are not ‘the real risk’. But this is misguided, for a number of reasons. First, the ‘credit VAR’ figure one would obtain from the method is up-to-date and responds to market changes immediately. Second, one may as well manage that number as opposed to one coming from historical simulation: the market-implied figure is likely to be more relevant, as well as free from such subjective biases as what data are ‘relevant’ to the calibration. The investment decisions that come from management of a market-implied VAR are, by presumption, the ones that the market considers most beneficial from a mark-to-market perspective: for example, chopping long positions in volatile credits. Third, if the presence of the risk premium were really deemed unsatisfactory, one could strip it out of the CDS quotes to start with and then rerun the analysis of this article using just the ‘default part of the spread’ as input – at least, in principle. The question of how much of the spread is real risk has been examined by some authors (see Berg & Kaserer, 2008, and references therein). However, identification of the risk premium is a matter of statistical estimation rather than mathematical proof, and it is usually accompanied by a fair amount of model risk. For example, Berg & Kaserer point out that, judging from default experience, only about one-third (20bp/57bp) of the investment-grade credit spread in 2003–08 could be attributed to default risk, with the rest being risk premium. Yet in 2008–09, where Itraxx Main has been out at 200bp, there have still been no defaults in that index. Does the one-third figure carry over, or should it now be lower, or higher? In using, for example, equity-based factor models to explain and quantify the expected default frequency (in real probability) to compare with the market-implied one, we have to implicitly assume that the parameterisation of the factor model is static over time. It could certainly be argued that not trying to extract the risk premium introduces fewer assumptions into the answer.

How does one construct a VAR model across credit asset classes? Assign to each credit an explanatory variable, which is the log firm value, denoted X here. The present value of any credit position can therefore be written as a function of X. The distribution of X is not normal, as it would be in a diffusion model, but can be calculated easily enough from the Arrow-Debreu prices. There exists a monotonic transformation g that makes a N(0, 1)-distributed random variable Z have the same distribution as X; this function is simply the composite of the inverse cumulative distribution function of X and the cumulative normal distribution function, and is referred to in Credit Suisse First Boston (2004) as the ‘transfer function’. If, for reasons of simplicity, we choose to model the correlation through the Gaussian copula, then we are effectively asserting that we believe the Z-variables for different issuers to have a joint distribution that is multivariate normal (as opposed to merely having normal marginals). The correlations between the explanatory variables have to be examined from historical credit spread data, but experience shows that the precise values of the spread correlations are of secondary importance in determining the VAR and the trading decisions necessary to manage risk. To parameterise spread correlations in a parsimonious way, we suggest allocating each issuer to an industrial sector, thereby giving it a ‘beta’ to that sector, and then providing a correlation matrix between sectors. This is obviously preferable to having to estimate a full 1,000 × 1,000 correlation matrix for a portfolio of 1,000 credits.

In equations, we have for each issuer (j):

$$X_j = g\left[\beta_j V_{d(j)} + \sqrt{1-\beta_j^2} z_j\right], \quad E[V_{d(j)}] = \rho_{j,k}$$

where the V’s are the sector factors, tj is the idiosyncratic return of the jth issuer, st(j) is the sector in which the jth issuer resides, and $\rho_{j,k}$ is the sector-sector correlation matrix. The transfer function g encodes the non-normality of the credit distribution, and hence the jump risk, and is obtained from the structural model without reference to historical data.

To simulate the present values of the individual credit positions, we first simulate the sectoral factors and the unsystematic returns, then the explanatory variables $X_j$, and thence the present value of the position can be found from the structural model. Note that this approach is identical to simulating losses in default/no-default models, only in that case the transfer function g(t) is simply a step function equal to one minus recovery if x is less than the default point, and zero otherwise. For a fuller discussion, see Credit Suisse First Boston (2004).

Another important benefit of this framework is that it identifies risk from curve positions. No two instruments of different maturity can statically hedge each other: the position exhibits negative convexity to large downward moves in the underlying firm value. The so-called DV01-neutral steepener trade, which consists of selling short-dated protection against longer-dated, is then seen to be potentially very risky. The concept of ‘credit delta’, which arises from a perception of the world in which credit spread curves move parallel 1bp at a time, is essentially another attempt to use interest rate models to risk-manage credit. When the monolines blew up, the players with this type of position may well have thought they
were delta-neutral or even slightly short, but suffered as the curves inverted. This kind of risk gets picked up by the Lévy structural model, because the probability of a ‘bad event’ is not vanishingly small and the associated mark-to-market loss is large.

**Conclusions and discussion**

We have demonstrated a structural model with jumps and shown that the market appears to price jump risk into the CDS curve. Outside risk management, applications include the valuation of gap options or out-of-the-money CDS swaptions, and the analysis of structures such as constant proportion debt obligations – which even in early 2007 looked sub-investment-grade in this model.

As pointed out by the referee, the model-implied VAR, or market-implied VAR, has potential applications on both the buy side and the sell side and in financial regulation. Certainly, anyone who is mark-to-market-sensitive needs to think about using this kind of model. This takes account of much of the financial world. However, even for those who consider themselves non-mark-to-market, information from the rest of the market about the perceived credit quality, or likelihood of deterioration of perceived credit quality, is surely important.

Furthermore, such investors may well not be immune to mark-to-market effects even if they choose not to observe fair-value accounting: a loan book may not be marked-to-market, but the parent firm may well have publicly traded equity and traded debt and CDSs. The debt spread of the firm, its future funding costs and also the equity price will be determined by the market’s perception of the quality of the assets on the balance sheet, and also (by appeal once more to the structural model) the volatility of those assets. With balance sheets being under more and more scrutiny, it is very unlikely that any market participant can declare himself, Canute-style, to be unaffected by market opinion.

As to the question of financial regulation, this is an area in which the present author has little direct experience, and clearly more work and testing needs to be put in before the ideas here could be injected into the regulatory environment. That said, we have enough experience of these type of models to make some general points. First, these models were run on two credit trading books throughout the credit crunch and the model 10-day 99% VAR level was never breached once, though it was never so high that such a breach would have been out of the question given the positions being run and the market conditions (for example, certain financial institutions going bankrupt in 2008). Second, when spreads are tight and not very volatile, the methods here are considerably more conservative, particularly at high percentiles, than historical VAR. (To take a slightly ridiculous extreme, it is theoretically possible for a particular spread not to move for a whole year, but clearly it could move in the future, and the model-implied answer gives some sort of indication of the probabilities.) Returning to the example in figure 4, a five-times levered five-year credit-linked note on that credit would have picked up 400bp carry but according to the model had roughly a 2% chance of being forcibly degeared in its first year. Use of a model such as this might well have prevented leverage from getting as high as it did. On the other hand, after a crisis, as market-implied volatilities decline and credit spreads tighten, it is possible for historical VAR to overestimate the risk that is being faced, thereby penalising a desk that is correctly trying to get long risk (having previously made the opposite mistake of providing no effective restraint when one was required), whereas the market-implied VAR tends to contract more rapidly as market conditions improve.

An interesting avenue for further research is the explanation of credit spread levels and, in particular, short-term spreads. It seems that there is a large risk premium built into short-term spreads, probably due to the potential mark-to-market consequences of being short CDS protection and being forced to unwind at a loss. In other words, the seller of a CDS is short a deep-out-of-the-money barrier option that requires a high premium because of the jumps. In such a model the observed CDS spread would be the default risk plus the forced-unwind option (here, it is as usual just the default-contingent part). The principal conclusion of this article can be made more provocative. Was the probability of ‘black swans’ really theoretically and practically impossible to estimate, as has so often been claimed? Or was it just lying beneath the surface, and easily detected with the structural model? ■

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10 Remember figure 4 is for one-month horizon

11 Longstaff, Mithal & Neis (2005) assert that “most of the credit spread can be explained by default”. That may have been true back in 2004–06, but now it seems questionable as mark-to-market risks have become more of a concern.