Valuation and risk analysis for Dutch pension schemes

This paper examines fair value liabilities and derives the implied indexation rate as the measure of the benefits provided by the conditional indexation mechanism. Further, it assesses the sensitivity of the fair value liabilities to real and nominal curves and discusses immunisation techniques derived from this framework

BY VLAD PUTYATIN AND STEFAN DE WACHTER

THERE HAVE BEEN SIGNIFICANT DEVELOPMENTS in the life and pensions industry over the last few years. In countries such as the UK, Denmark and the Netherlands, the regulatory framework has shifted from book valuation towards fair valuation of both assets and liabilities. Traditionally, asset management solutions for pension schemes were targeting long-term objectives such as full or partial indexation of liabilities and reduction of the contribution rate for the sponsor. Simultaneously, short-term mark-to-market exposure of a pension plan was smoothed out over time. The smoothing technique was promoted by regulators as well as by accounting standards of that period. Recent advances in the regulatory framework forced pension schemes to start developing LDI strategies that satisfy regulatory constraints and meet the long-term objectives of pension schemes. An overview of the conditional indexation methodology and market strategies can be found in Kocken et al. (2005).

In the current environment, regulatory constraints consist, broadly speaking, of two components. In the short term, pension funds are only required to maintain sufficient reserves to absorb one year's typical extreme market moves. In particular, there are no reserve requirements for the conditional part of liabilities, that is, those based on future indexation. In the long run, however, in accordance with the continuity analysis imposed by the pension regulator, the fund has to consider various instruments that facilitate inflation-indexation of nominal liabilities.

Although an overlay of nominal swaps can provide a short-term immunisation against rate fluctuations, this solution will not be consistent with the regulator's long-term indexation ambitions. These ambitions can reasonably be expected to be met by a traditional asset allocation strategy consisting of the investment into fixed-income assets, equities and real-estate. This asset allocation, however, can lead to a significant asset liability mismatch incurred in the short term, leading to a growing pressure on the plan sponsor to come up with an alternative investment strategy that protects solvency against unfavourable market scenarios.

The aim of this article is threefold. Firstly, we develop a mathematical framework to determine the market value of liabilities, including the fair valuation of the conditional indexation option. This enables a pension fund to determine the amount of inflation indexation that pension holders are locking into. Our approach to finding the implied indexation is

Figure 1. The fair value of liabilities as a function of the coverage ratio for different breakeven inflation scenarios

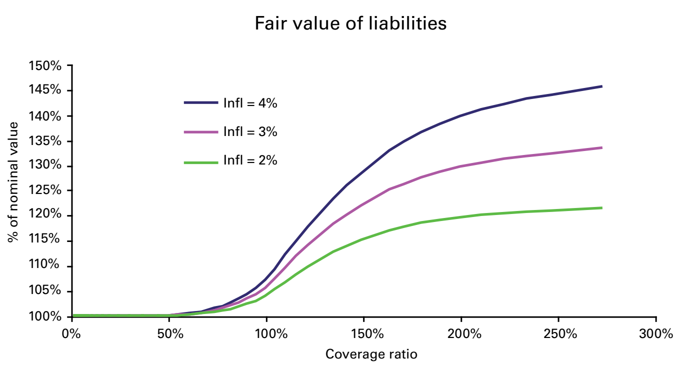
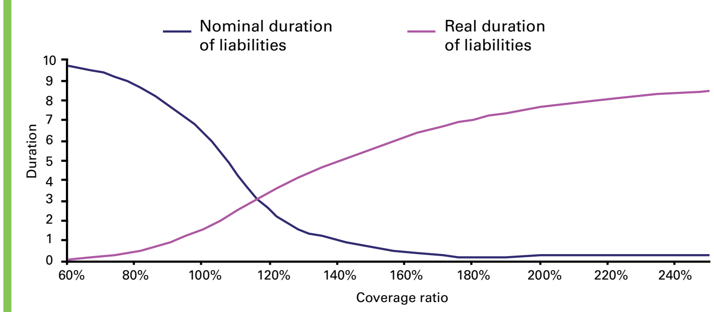


Figure 2. Nominal and real durations of fair value liabilities as a function of coverage ratio



based on risk neutral valuation principle, however, the model can be adapted to project liabilities under the actual probability measure.

Secondly, we value a default put option on assets of the pension fund struck at liabilities. Effectively, this approach enables the sponsor to assess the riskiness of various asset allocation strategies, since the value of the put option is akin to the insurance against a shortfall (i.e. present value of the probability weighted injections from the sponsor). This option is often used in corporate finance to assess contribution and payouts of various stakeholders involved. In the context of pension funds, the regulator's requirement is to hold buffer capital to absorb adverse market moves. In case of a shortfall below the trigger level of 105%, the pension fund would have to present a recovery plan designed to restore solvency back within three years. Potentially this solution could involve an increased proportion of nominal matching to prevent further erosion of the capital buffer. Although pension plans are not recognised on a company's balance sheet under Dutch GAAP, the advent of the IFRS accounting standards for multinationals (or RJ271 for unlisted companies) can change this going forward. In any case, knowing the true cost of hedging the tail risk can be advantageous when it comes to the valuation of various conditional indexation mechanisms and the development of robust asset allocation strategies that minimise buffer requirements for pension schemes.

Finally, we will discuss the construction of the replicating portfolio. The goal of this analysis is to understand sensitivities of the fair value liabilities to real and nominal curves based on the coverage ratio at any point in time. This should also give us an idea of the suitability of potential asset allocation strategies. Often portfolio managers have to guess the amount of indexation incorporated in pension liabilities based on the spot coverage ratio. The model developed in this article provides a more reliable way of assessing sensitivities of the fair value liabilities. If a pension fund decides to invest a portion of the portfolio into real and nominal instruments directly, the mismatch between fair value liabilities and the asset allocation can be reduced when compared to traditional asset allocation. At the same time, however, because of market imperfections, incompleteness (e.g., inability to directly trade Dutch wage inflation) and feedback effects of large order executions, we do not advocate exact immunisation strategies.

The paper is organised as follows. In the following section we develop an analytical framework that enables us to value the indexation option embedded in liabilities. We then discuss implications of our analysis for a typical asset allocation in Holland under various market conditions. An alternative modelling methodology based on multistage stochastic programming can be found in Klein Haneveld et al. (2006).

Modelling Methodology

The goal of the fair-value methodology is to consider liabilities as a contingent claim and obtain a replicating portfolio that matches liabilities at a fixed maturity date T. Nominal liabilities are set at the inception of the policy and are denoted by $L(0,T)$. They represent nominal (undiscounted) cash flows due at maturity date T, without taking any future inflation uplifts. As time t evolves, nominal liabilities $L(t,T)$ are ratcheted up in accordance with the realised inflation and the coverage ratio[1] of the company. Note $L(t,T)$ represents the nominal value at time T of the liabilities, given information accrued until t. In other words, $L(t,T)$ is similar to the notional of a zero-coupon inflation-linked bond. $L(0,T)$ is the notional of the bond, while $L(T,T)$ is the pay-out based on realised inflation. It is well-known that the market value of an inflation linked bond at time t is equal to $L(t,T)$ discounted by the real rate curve. Alternatively, we can inflate $L(t,T)$ at the breakeven inflation rate till maturity to obtain $L(T,T)$ and apply the nominal discount factor back to time t. In our case, however, the indexation payout is conditional on the coverage ratio as well, which makes pricing less straightforward than in the case of a zero-coupon inflation-linked bond. Despite this apparent complexity, we can derive an elegant pricing equation using the symmetry of the underlying system that we discuss below.

To proceed further, we introduce the following processes for the reference asset portfolio $A(t)$ and the short rate $r(t)$. We assume that the asset portfolio is based on a typical asset allocation, which is driven by a geometric Brownian motion. For simplicity, we use a simple single-factor Ho-Lee model for the nominal rate.[2] Under the risk-neutral pricing measure Q we

1 The coverage ratio is the ratio of the market value of assets to discounted value of nominal liabilities (without taking future indexation into account), i.e., a measure of solvency.

2 As will be apparent later, the choice of the interest rate model is not essential as we will introduce a forward measure to simplify the pricing equation.

pose the following system of SDEs for the reference portfolio, the nominal rate and the liabilities, respectively:

1
$$\frac{dA(t)}{A(t)} = r_t dt + \sigma_A dW_t^A$$
$$dr(t) = (\mu - \lambda\sigma_r)dt + \sigma_r dW_t^r$$
$$dL(t,T) = f(A,L,r)dt$$

where λ is the market price of interest rate risk and f is an indexation rule defined below. The standard Brownian motions W^A and W^r have correlation ρ; while σ_A and σ_r are volatilities of the reference portfolio and short rates, respectively.

The nominal coverage ratio $x(t)$ is defined as the ratio of assets to discounted value of nominal liabilities, i.e.,

2
$$x(t) = \frac{A(t)}{L(t,T)Z(t,T)}$$

where $Z(t, T)$ is the price of a nominal zero coupon bond[3] under the Ho-Lee framework, i.e.,

$$Z(t,T) = \exp\left\{-(T-t)r_t - (\mu - \lambda\sigma_r)(T-t)^2/2 + \sigma_r^2(T-t)^3/6\right\}$$

Note that the denominator of the coverage ratio (2) is the value of nominal liabilities at time t expressed in present value terms, assuming no further inflation indexation adjustments are made between t and T.

Typically, the conditional indexation rule f is defined in terms of the coverage ratio. When the coverage ratio drops below a certain threshold, no or only partial inflation indexation is paid. When the coverage ratio is high, full inflation indexation is delivered. In other words, f provides a type of smoothing mechanism that allows pension schemes to pay inflation when markets performed well and reduce payments in alternative circumstances. Mathematically, the indexation rule f has a floor at zero, is capped by realised inflation, and is otherwise linear in the coverage ratio, starting from the level $1+\beta$:

$$f(A,L,r,t,T) = L\min\left\{r_I, k\max\left\{\frac{A}{LZ(t,T)} - (1+\beta), 0\right\}\right\}$$

where k denotes the steepness of the indexation rule and defines the upper indexation threshold. The parameter β represents the indexation trigger level and is typically set by FTK regulatory tests. The level reflects the degree of the asset and liability mismatch as well as certain actuarial risks. In general, a less risky asset allocation with respect to nominal liabilities would command lower margin β and vice-versa. In our numerical calculations in the following section, we set β at the minimal level possible of 5%.

The (breakeven) inflation rate r_I is constant in our model. Ideally, it needs to be represented as another stochastic process. Here, however, we decide in favour of model simplicity, since there are other non-quantitative regulatory aspects that can have an impact of a similar order of magnitude and can not be easily embedded into a quantitative model (e.g., uncertainty regarding a change in the regulatory framework itself). We treat r_I as a parameter in our model and stress test liabilities with respect to this parameter at a later stage to construct an efficient asset allocation scheme. This is similar to using constant parameters in the Black-Scholes model like volatility and rates, which are stressed to produce Vega and Rho sensitivities in order to adjust risk exposures on trading books, for instance.

We now employ the risk-neutral valuation principle and Ito's lemma to obtain the pricing equation. Applications of risk-neutral valuation principle to insurance and pension liabilities have been extensively studied by Grosen and Jorgensen (2000), Prieul et al. (2001) and others. We obtain the following three-dimensional parabolic PDE for fair-value liabilities V:

3
$$\frac{\partial V}{\partial t} + rA\frac{\partial V}{\partial A} + (\mu - \lambda\sigma_r)\frac{\partial V}{\partial r} + \frac{1}{2}\sigma_A^2 A^2 \frac{\partial^2 V}{\partial A^2} +$$
$$\rho\sigma_r\sigma_A A\frac{\partial^2 V}{\partial A^2} + \frac{1}{2}\sigma_r^2\frac{\partial^2 V}{\partial r^2} + \frac{\partial V}{\partial L} f\left(\frac{A}{LZ(t,T)}\right)L - rV = 0$$
$$V(A,L,r,T) \equiv L$$
$$V(A,L,r,t) \approx LZ(t,T), \quad A/LZ(t,T) << 1$$
$$V(A,L,r,t) \approx L\exp(r_I(T-t))Z(t,T), \quad A/LZ(t,T) >> 1$$

The boundary conditions stipulate that (i) at maturity liabilities are equal to L; (ii) for lower coverage levels liabilities are not accrued going forward and are close to discounted value of L at any point in time, (iii) for higher coverage ratios liabilities are accrued at the inflation rate over time. In fact, in case of higher coverage ratios the value of liabilities is simply given by the notional $L(t,T)$ accrued at the breakeven inflation rate and discounted with the nominal discount factor. This is exactly the same pricing formula as used to price a conventional zero-coupon inflation-linked bond.

Similarly, in order to price a default put $P=P(A,L,r,T)$ on assets of the company truck at liabilities, the following set of boundary conditions should be used:

4
$$P(A,L,r,T) \equiv \max(A-L,0)$$
$$P(A,L,r,t) \approx A - LZ(t,T), \quad A/LZ(t,T) << 1$$
$$P(A,L,r,t) \approx 0, \quad A/LZ(t,T) >> 1$$

Equations (3) and (3) with boundary conditions (4) are difficult to solve outright. However, they admit a similarity solution that reduces the dimension of the pricing equation from three to one. The substitution below is akin to the introduction of a new numeraire given by the discounted value of liabilities:

5
$$V(A,L,r,t) = LZ(t,T)u\left(\frac{A}{LZ(t,T)}, t\right)$$

where $u=u(x,t)$ depends on the coverage ratio and time since inception only. Plugging (5) into (3), we obtain the following parabolic PDE for $u(x,t)$:

6
$$\frac{\partial u}{\partial t} - xf(x)\frac{\partial u}{\partial x} + \frac{1}{2}\left(\sigma_A^2 + 2\rho\sigma_A\sigma_r(T-t) + \sigma_r^2(T-t)^2\right)x^2\frac{\partial^2 u}{\partial x^2} + f(x)u = 0$$
$$u(x,T) \equiv 1$$
$$u(x,t) \approx 1, \quad x << 1$$
$$u(x,t) \approx \exp(r_I(T-t)), \quad x >> 1$$

3 For completeness, the value for $Z(t,T)$ can be also derived from the pricing equation to be introduced later.

Similarly, for boundary conditions (4) for the default put option, we obtain the following set of transformed boundary conditions (here we introduce notation $q=q(x,t)$ for the transformed put option value P):

$$q(x,T) \equiv \max(1-x,0)$$
$$q(x,t) \approx 1-x, \quad x << 1$$
$$q(x,t) \approx 0, \quad x >> 1$$

The underlying pricing equation remains as in (6).

In the introduction we mentioned that portfolio volatility is the only relevant factor for pricing the highly structured liabilities $V=V(A,L,r,t)$ (or $u=u(x,t)$ in terms of the transformed variables). By this we meant the volatility term in the pricing equation (6) which is based on portfolio volatility, nominal rate volatility and their mutual correlation term ρ.

If, instead of the Ho-Lee model, we were to choose a more common three-factor model for the dynamics of nominal interest rates, we would have obtained the same pricing equation for $u=u(x,t)$, but with a different volatility term only (which would have aggregated volatility impact of all three factors, their mutual correlations and correlations with the index). Also, we can always treat breakeven inflation r_I as another stochastic factor and apply the same change of the numeraire. This will lead to a two-dimensional problem for pricing liabilities.

The linear partial differential equation (6) can be solved in a spreadsheet by implementing a simple backward induction algorithm, i.e., a trinomial tree with the boundary conditions. Alternatively, one could use the Crank-Nicolson scheme to achieve a more efficient implementation. From a stability perspective, it is best to introduce log variables for x beforehand.

For the case of a constant indexation methodology, i.e., $f(x)=r_I$, the PDE admits an obvious solution $u(x,t)=\exp(r_I(T-t))$, which represents the indexation of the notional with respect to the breakeven inflation. By applying transformation (5) we obtain the standard pricing formula for a zero-coupon inflation-linked bond:

$$V(A,L,r,t) = L(t,T)\exp(r_I(T-t))Z(t,T)$$

The pricing equation implies that the coverage ratio x defined by (2) follows a random walk driven by the following SDE:

$$dx/x = -f(x)dt + \left(\sigma_A^2 + 2\rho\sigma_A\sigma_r(T-t) + \sigma_r^2(T-t)^2\right)dW_t$$

This equation, together with the fact that the accrual rate to be used for the valuation of liabilities is $f(x)$, provides an alternative way of solving the valuation problem. One can simply generate a binomial tree for x and accrue a terminal value of 1 using rate $f(x)$. This binomial tree methodology yields the same result for liabilities as solving the PDE for the coverage ratio x numerically.

Similarly, to value a put option on assets struck at liabilities (the default put), one can diffuse the payoff max $(1-x, 0)$ backwards on the binomial tree using the accrual rate $f(x)$. It is best to apply boundary conditions in this case as they enable one to obtain the fair value of liabilities for different values of the coverage ratio and derive the sensitivity factors discussed below at the same time.

The fair value model developed in this section is analogous to firm value models pioneered by Merton (1974) and developed further by Leland and Toft (1996). Here we apply this type of models to a pension fund with highly structured liabilities, rather than to the valuation of corporate debt.

In Figure 1 we illustrate the fair-value of liabilities V for a typical asset allocation (as discussed later) as a function of the initial coverage ratio for different inflation levels. In order to find allocations between various funds, we calculate the following key sensitivities of the fair value of liabilities V based on our numerical solution. These key sensitivities (deltas) are also needed to replicate the fair value liabilities in practice. In the notation below, when the sensitivity to inflation is calculated the nominal rate and the index are kept unchanged[4] and vice versa. With these conventions, the key sensitivities to nominal and inflation rate are given by:

$$\frac{1}{V}\left(\frac{\partial V}{\partial r_I}\right)_{r_{nom},\ A} = \frac{1}{u}\frac{\partial u}{\partial r_I}(x,T,r_I)$$
$$\frac{1}{V}\left(\frac{\partial V}{\partial r_{nom}}\right)_{r_I,\ A} = -(T-t)\left(1-x\frac{\partial u}{\partial x}\right)$$

We added a sub-index to nominal rates (i.e., $r \equiv r_{nom}$) to emphasise that we deal with the nominal rate, as opposed to the inflation rate r_I and real rate r_{real} introduced below.

From the replication point of view it may be more practical to choose real and nominal rates as independent variables to replicate liabilities, since there are tradable instruments in the market that justify this choice of the notation. This yields the following key sensitivities:

$$D_{real}^{Liab} = -\frac{1}{V}\left(\frac{\partial V}{\partial r_{real}}\right)_{r_{nom},\ A} = \frac{1}{V}\left(\frac{\partial V}{\partial r_I}\right)_{r_{nom},\ A}$$
$$D_{nom}^{Liab} = -\frac{1}{V}\left(\frac{\partial V}{\partial r_{nom}}\right)_{r_{real},\ A} = -\frac{1}{V}\left(\frac{\partial V}{\partial r_{nom}}\right)_{r_I,\ A} - \frac{1}{V}\left(\frac{\partial V}{\partial r_I}\right)_{r_{nom},\ A}$$

where D_{real}^{Liab} and D_{nom}^{Liab} denote real and nominal duration of the liabilities, respectively. The definition of real duration reflects the following curve scenario: real rates move up, while inflation rate moves down to compensate for the move in real rates, so that nominal rates stay unchanged across the curve, i.e., a productivity increase in a very competitive economy. The definition of nominal duration D_{nom}^{Liab} reflects a shift in nominal rates driven by the inflation rate (while real rates stay unchanged across the curve), i.e., an increase in inflation expectations. The real and nominal duration profiles are illustrated in Figure 2. A low coverage ratio implies a higher allocation to nominal bonds, while a higher coverage ratio favours a higher allocation to real bonds.

There is a simple expression for the sensitivity of liabilities with respect to index A:

$$\Delta = \left(\frac{\partial V}{\partial A}\right)_{r_I,\ r_{nom}} = \frac{\partial u}{\partial x}(x,T,r_I)$$

The indexation option is akin to a call spread.

Results and Discussions

Pricing liabilities and the implied indexation rate

A typical asset allocation in Holland has approximately the following composition: 65% fixed-income, 28% equity and 7% real estate. As an example, consider a pension fund with nominal discounted value of lia-

4 Sub-indices outside the bracket are kept fixed when the derivative is taken.

bilities $L(0,T)Z(0,T)$=100 and coverage ratio $x(0)$=125%. Assume for simplicity that liabilities are of 10-year maturity. We adopt the following indexation rule: the indexation policy begins when the coverage ratio is above 105% and increases in a linear fashion until the upper threshold for the coverage ratio of 130% is reached. There is no indexation below the 105% level, while at 130% and above the indexation payout is capped by realised inflation over the period. We assume that breakeven inflation rate over this period of time is r_I = 4%. From the pricing model, we find that the current fair value of the payout for policyholders is 117.78 in this case (i.e. projected benefit obligations expressed in present value terms). This implies an unconditional indexation rate of 1.64% over the next 10 years. In other words, in the context of financial markets we could consider a swap where the conditional indexation is swapped against a fixed rate of 1.64% over the next 10 years. Such a swap would be at par today. Although the indexation mechanism can be compared to a fixed payment on liabilities over time in terms of the expected payout, the former is less risky for a pension fund. This is achieved due to the smoothing mechanism provided by the conditional indexation. Our model enables pension funds to quantify the benefits of the conditional indexation through the valuation of the put option on assets of the company struck at the conditionally indexed liabilities. In our case, we find that the cost of the put option that protects the solvency against adverse market moves is 12.33. If instead the company were to use fixed rate indexation based on the 'equivalent' rate of 1.64%, the cost of the default put would rise to 16.93. This indicates the benefits of the conditional smoothing mechanism that helps to minimise reserving requirements relative to the plain indexation rate.

Risk assessment of the indexation policy

We now illustrate the sensitivity of the payout and put valuation to the indexation strategy $f(.)$. To achieve this, we vary the upper indexation threshold by 5% from the current level of 130%. We summarise the results in Table 1 below.

Table 1: Fair value of liabilities, implied inflation and put values for various levels of the upper indexation threshold, assuming a lower threshold of 105%

Upper Threshold	Policy	Implied Inflation	Put
120%	120.74	1.88%	13.50
125%	119.44	1.78%	12.99
130%	117.78	1.64%	12.33
135%	116.60	1.54%	11.86
140%	115.72	1.46%	11.51

The higher the indexation threshold, the stronger the coverage ratio should be before policyholders start receiving full realised inflation. Hence, policyholders' payout will be lower on average in this case, and so should be the value of the put because of a greater smoothing advantage. When the upper indexation threshold increases by 1%, the fair value of the policy decreases by 0.2%. In response, the value of the put option decreases by 0.76%.

Similarly, we establish the sensitivity of the policy payout and put values to the lower indexation limit. Currently it is set at 105%. We increase this level in 5% increments, while keeping the upper limit constant at 130%. The coverage ratio is kept unchanged at 125%. The results are shown in Table 2 below.

Table 2: Fair value of liabilities, implied inflation and put values for various levels of the lower indexation threshold, assuming a upper threshold of 130%

Lower Threshold	Policy	Implied Inflation	Put
105%	117.78	1.64%	12.33
110%	116.88	1.56%	11.97
115%	115.31	1.42%	11.35
120%	111.82	1.12%	9.99
125%	108.57	0.82%	8.74

Portfolio managers can use these results to calculate the benefits of the indexation for their exact indexation rule in the following way (using the tables as partial derivatives with respect to lower and upper threshold levels). Suppose we would like to calculate indexation based on a [110%, 125%] rule. We know that implied inflation is 1.64% based on the [105%, 130%] rule. By moving the lower threshold from 105% to 110%, implied inflation reduces by 8 basis points (see Table 2). Likewise, by reducing the upper threshold from 130% to 125%, indexation increases by 14bp (see Table 1). The aggregate effect is a 6bp increase towards 1.70% for [110%, 125%] rule.

In our study the policy value and the implied indexation inflation are obtained from the risk-neutral pricing model. Hence, the policy valuation does not incorporate any risk premium. The pension fund would need to assess whether the gap between the breakeven inflation and the implied inflation indexation can be safely bridged by the extra return on equity and real estate. Since our holdings in these asset classes are only 35% of the total asset allocation, it is unclear that this will be achieved in a high inflation environment.

Sensitivity of fair value liabilities to real and nominal rates

Our model enables us to decompose interest rate sensitivity of liabilities into real and nominal components. In our example, for a coverage ratio of 125% and maturity of liabilities of 10 years, we obtain real duration of liabilities of four years and nominal duration of two years. The rest of the sensitivity of liabilities comes from exposure to other asset classes (equity and real estate in our example).

For lower coverage ratios (e.g. 110%) nominal duration of liabilities rises to six years, while real duration declines to two years. As expected, there is a clear shift towards a higher proportion of assets allocated to nominal bonds to protect the solvency from deteriorating.

In contrast, for high coverage ratios, say 140%, nominal duration of liabilities will be just around one year, while real duration will increase to five years. As the coverage ratio increases further, the proportion of nominal liabilities will be close to zero, while real duration increases gradually to 10 years. The speed of this convergence is logarithmic in the coverage ratio. For example, in order to reach nine years of real duration, the coverage ratio should be as high as 270%.

Hedging interest rate risk

The advent of the new regulatory framework based on market value principles has paved the way for partial risk hedging. We think immunisation of nominal and real interest rate risk is a good starting point.[5] The pricing methodology we developed in the previous section enables us to assess real and nominal duration exposures. To use these sensitivities in a meaningful way for the purpose of adjusting the asset allocation, we need to make sure that the portfolio volatility that we used for pricing does not change when we adjust our asset allocation, i.e., the volatility term in equation (6) should be kept constant. This implies that the original asset portfolio is replaced by a new investment portfolio provided that it does not change the volatility term in (6).[6] The nominal and real components of the new portfolio will be determined from our pricing model based on the original allocation.

In Figure 1 we illustrate sensitivity of fair-value liabilities to various values of the breakeven inflation rate. This sensitivity increases as the coverage ratio gets larger. Figure 2 shows the duration of the fair value of liabilities with respect to the real and nominal interest rate. When the coverage ratio is low, liabilities behave like a nominal bond. As the coverage ratio improves, indexation is ramped up, leading to greater sensitivity to the real rate. Note that the variability of these measures is greatest around the level of coverage observed in the current market environment. This makes the case for precise mathematical modelling very compelling.

Conclusion

In this article we have analysed LDI strategies based on modelling liabilities as a contingent claim. This approach should enable pension funds to determine the amount of inflation indexation embedded in fair value liabilities. To assess the true costs of hedging the tail risk and riskiness of various indexation rules we value a default put option. This metric will facilitate the development of robust asset allocation strategies that minimise buffer requirements for pension schemes. In order to reduce the riskiness of asset allocation strategies we also assess the sensitivities of the fair value liabilities to real and nominal curves. In the higher inflation environment that we may experience in the future, these may be the key variables affecting the solvency of the pension scheme.

To conclude, we remark that a similar model can be readily applied to UK with-profits schemes, as they enjoy the reversionary bonus principle, which can be thought of as some form of an indexation. Future work should include analysis of the aggregate balance sheet for the pension plan and the corporate sponsor to identify an optimal asset allocation and exploit synergies between the two.

5 We do not advocate full immunisation of liabilities.
6 For a detailed discussion on how the volatility terms in (6) can be kept constant, we refer the reader to the Appendix.

Vlad Putyatin and Stefan de Wachter are portfolio managers at PIMCO Europe. Email: vlad.putyatin@pimco.com, stefan.dewachter@pimco.com

The authors gratefully acknowledge detailed comments by two anonymous referees, which greatly improved the paper. This article contains the current opinions of the authors only and not of PIMCO or any of its subsidiaries. The article is intended for educational purposes only and should not be considered as investment advice or a recommendation of any particular security, strategy or investment product. Information contained herein has been obtained from sources believed to be reliable, but not guaranteed.
See www.life-pensions.com for appendix to this article

References

Grosen, A. and Jorgensen, P.L.,
Fair valuation of life insurance liabilities: the impact of interest rate guarantees, surrender options and bonus policies,
Insurance: Mathematics and Economics, 26, 37-57 (2000

Kocken, T., J. van der Hoek, A. Michaels and D. Bieber,
From Conditional to Full Indexation: A Discussion of Liability Solutions Used in Dutch and UK Pension Funds,
in: Inflation-linked Products, Opportunities for Portfolio, Asset & Liability Managers, RISK Publications (2005)

Prieul D., Putyatin V., Nassar, T.,
On Pricing and Reserving With-Profits Life Insurance Contracts,
Applied Mathematical Finance 8, 145-166 (2001)

Klein Haneveld, W., Streutker, M., Van der Vlerk, M.,
ALM Modelling for Dutch Pension Funds in an Era of Pension Reform,
Operations Research Proceedings 2006, Springer Berlin Heidelberg (2006)

Leland, H. and Toft, K.,
Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads,
Journal of Finance, v51(3 Jul), 987-1019 (1996)

Merton, R.,
On the Pricing of Corporate Debt: The Risk Structure of Interest Rates,
Journal of Finance v 29(2), 449-470 (1974)

Submission guidelines for technical articles

Life & Pensions welcomes the submission of technical articles on topics relevant to our practitioner readership. Core areas include solvency and economic capital modelling, the measurement and management of financial, biometric and operational risks, market-consistent valuation and financing of life and pension balance sheets and cashflows, and investment management. This list is not an exhaustive one.

The most important publication criteria are originality, exclusivity and relevance. In the interests of our readers, we attempt to strike a balance between these. Thus, while we will not publish executive summaries of longer papers (on the grounds of exclusivity), we may accept papers that draw partially but not completely on research submitted elsewhere should our referees recommend it on the grounds of originality and relevance to practitioners.

Given that *Life & Pensions* technical articles are shorter than those in dedicated academic journals, clarity of exposition is another yardstick for publication. Once received by the editor and his team, submissions are logged, and checked against the criteria above. Articles that fail to meet the criteria are rejected at this stage. Articles are then sent to one or more anonymous referees for peer review. Our referees are drawn from the actuarial, risk management, treasury and investment departments of major life and pensions companies, in addition to academia and regulatory bodies. Depending on the feedback from referees, the technical editor makes a decision to reject or accept the submitted article. His decision is final.

We also welcome the submission of brief communications. These are also peer-reviewed contributions to *Life & Pensions* but the process is less formal than for full-length technical articles. Typically, brief communications address an extension or implementation issue arising from a full-length article that while satisfying our originality, exclusivity and relevance requirements, does not deserve full-length treatment.

Submissions should be sent to the editor at *technical@incisivemedia.com.* The preferred format is MS Word, although Adobe PDFs are acceptable. The maximum recommended length for articles is 3,500 words, and for brief communications 1,000 words, with some allowance for charts and/or formulas. We expect all articles and communications to contain references to previous literature. We reserve the right to cut accepted articles to satisfy production considerations. Authors should allow four to eight weeks for the refereeing process.