Understanding longevity bonds

One of the major concerns of pension funds is the management of longevity risk. A new financial-actuarial asset called the longevity bond seems to present a solution to this problem. We investigate the power of such an instrument and show the role it can play in a pension fund portfolio

BY FRANCESCO MENONCIN

Introduction

In November 2004, the European Investment Bank (EIB) unveiled plans to issue the first longevity bond aimed at offering a partial longevity risk hedge to UK pension schemes and life insurers. Unfortunately, to our knowledge, the longevity bond has been first actively marketed but then withdrawn because of a lack of demand (some possible explanations are listed in Cairns et al., 2005).

The (25-year time to maturity) longevity bond was structured with coupons given by the product between a fixed amount of money (£50,000,000 for the whole issue) and the cumulative realised survival rate measured on an English and Welsh cohort of males aged 65 in 2003. Table 1 shows an example[1] where the cumulative survival rate is computed as the product of all the previous survival rates (for instance, in 2007, 96.345% is obtained through 0.99 × 0.988 × 0.985).

The correlation between the (cumulative) survival rate and the longevity bond coupons (see Table 1) makes the longevity bond a suitable asset for hedging against the so-called longevity risk, which consists of experiencing a survival rate higher than the prospective one. Even if the hedging role that the longevity bond can play for both pension funds and insurances is evident, it can be quite difficult to determine the exact amount of the bond that must be optimally held in a portfolio. Despite this difficulty, Menoncin (2006) demonstrates, in a quite general framework, what the role of a longevity bond is in the optimal portfolio of a representative consumer-investor. In what follows, we are going to look at the main financial intuitions behind those technical results. Finally, we will present a numerical simulation allowing us to understand whether a longevity bond plays a relevant role in an optimal portfolio.

Equilibrium and risks for a pension fund

When a worker subscribes to a pension fund, an actuarial fairness condition asks for the expected present value of all the future contributions paid by the worker to the fund to be equal to the expected present value of all the future pensions the fund will pay to the worker-pensioner. If this equality were not satisfied, it wouldn't be convenient for one of the two parties to sign the contract.[2] It can be shown (as in Battocchio et al., 2005) that such an equality has the following algebraic form:

$$\mathbb{E}_0^{\mathbb{Q}}\left[\int_0^{T_R}\left({}_s p_x\right)C(s)\upsilon(0,s)ds\right]=\mathbb{E}_0^{\mathbb{Q}}\left[\int_{T_R}^{\infty}\left({}_s p_x\right)P(s)\upsilon(0,s)ds\right] \quad (1)$$

where:

- $\mathbb{E}_0^{\mathbb{Q}}$ is the expected value operator under the information set available at time 0 and computed in a risk-neutral framework (i.e. under the so-called risk neutral probability $\mathbb{Q}$).
- T_R is the retirement date.
- $({}_s p_x)$ is the probability that an agent aged x survives for s periods until $x+s$ (for technical details on how to model a stochastic force of mortality, see Dahl, 2004).
- $\upsilon(t, s)$ is the discount factor from t to s (always positive and smaller than 1) computed at the instantaneously riskless interest rate $r(t)$. For the purpose of estimation, Cairns et al. (2005) show that such an interest rate should coincide with the EIB spot rate for the short-dated cash flows and with something over Libor-35 for longer-dated cash flows.
- $C(t)$ and $P(t)$ are contributions and pensions respectively; the sponsor decides either the contributions (in a defined contribution (DC) scheme) or the pensions (in a defined-benefit (DB) scheme) while the fund decides the other variable so that Equation (1) holds.

One topic is worth further investigation – one of the following conditions must happen:

- The financial market is complete (that is, there exists only one risk-neutral probability $\mathbb{Q}$). This means that any risk on the financial market can be hedged by means of a suitable portfolio (the hedging effectiveness positively depends on the correlation degree between the financial assets and the risk that must be hedged).

1 Indeed, the longevity bond coupon is computed on the actual survival rate of the reference population.
2 An identical fairness condition is introduced in Hau (2005). The departure from the actuarial fairness condition and all the possible explanations of it are studied in Miles and Cerny (2001)

Table 1: Coupon structure of a longevity bond			
Year	2005	2006	2007
Mortality rate	1%	1.2%	1.5%
Survival rate	99%	98.8%	98.5%
Cumulative survival rate	99%	97.812%	96.345%
Coupon (on £1,000)	990	978.12	963.45

Source: Azzopardi (2005)

- The financial market is incomplete (that is, there exists an infinite number of probability $\mathbb{Q}$). This is the most relevant case in practice but also the most difficult to handle from a mathematical point of view. Since the probability $\mathbb{Q}$ is not unique, then the arbitrage free prices on the financial market are not unique. Which probability an investor should choose for computing the asset values is one of the most challenging topic in today's finance. It can be demonstrated (see, for instance, Stoikov and Zariphopoulou, 2005) that an agent having a constant relative risk aversion optimally chooses the so-called minimax martingale measure.

The general results we are going to present are valid for both a completeand an incomplete market. The only difference is that in an incomplete market we are not able to carry out the computations for obtaining the optimal portfolio in a closed form (even if we are able to investigate the main behaviour of it). Accordingly, the numerical simulation presented later in this report describes a complete financial market.

From Equation (1) it is evident that the equilibrium condition for a pension fund is strongly affected by the following risks:

- Interest rate risk. Let us think of the case of a DB pension fund receiving contributions during a decreasing interest rate period. Since the collected money can be invested at interest rates which are lower, it becomes more difficult to guarantee the pensions the fund has committed to pay.
- Inflation risk. If the pension fund collects contributions during a high inflation period and pays pensions during a low inflation period, then it is paying back high-value money while it received low-value money.
- Mortality risk. When the people who subscribed to the pension fund start receiving their pensions, the longer their life, the longer the pensions must be paid. This means that any unexpected increase in the longevity coincides with a higher risk to go into lack of funds for paying pensions. This risk can be measured as the unexpected lower decreasing ratio in the survival probability $({}_tp_x)$ which is, of course, a stochastic variable itself.

The interest rate risk can simply be managed by using a suitable portfolio of bonds which have a strong negative correlation with the interest rate risk. Their main role, in fact, is to hedge the portfolio against the unexpected changes in the interest rate curve. In particular, it can be demonstrated that the bonds must be held in portfolio proportionally to their semi-elasticity to the interest rate (so called duration).

The inflation risk is more difficult to hedge since the financial market generally lacks assets that have a high correlation with the consumption price index. The inflation indexed assets play a crucial role in this regard.

Finally, the main problem about the mortality risk is that its source of randomness has a low correlation with the random variables on the financial market. In other words, there is a low correlation between the changes in the financial asset prices and the changes in the survival probability. Accordingly, any attempt to hedge the mortality risk with the 'usual' financial instruments would lead to a very poor result.

When an asset like the longevity bond is issued, the correlation between the asset prices and the force of mortality is 'artificially' strengthened and a hedging strategy can be more effective.

The longevity bond

In an arbitrage-free financial market the value of any financial asset coincides with the expected present value of all its future cash flows, discounted by the riskless interest rate under the risk neutral probability (which is not unique if the market is not complete). The longevity bond is an amortising bond whose face value is paid back in together with the coupons. Accordingly, the price of a longevity bond in t expiring in T can be written as

$$L(t,T) = \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^T \left({}_{s-t}p_{x+s}\right) v(t,s)ds\right] \quad (2)$$

When the survival rate of the pension fund subscribers $({}_{s-t}p_{x+s})$ remains unexpectedly high, the longevity bond coupons also remain high and it is easier for a pension fund to face the commitment to pay pensions.

It is important to highlight that the hedge we refer to can be more effective if the longevity bond population is closer to the population subscribing the pension fund. If this is not the case (so-called basis risk), then the hedging power of the longevity bond is reduced.

The longevity bond in an optimal portfolio

Given that the longevity bond can play a crucial role in hedging a pension fund portfolio, we still have to investigate which amount of longevity bond must be optimally held.

If we can invest in stocks, (ordinary) bonds and liquidity, it is easy to understand that the introduction of a longevity bond will mainly affect the weight of the asset with the highest correlation. It is evident that the expected present value of all future pensions, in the right-hand side of Equation (1), and the value of the longevity bond in Equation (2) are positively (and strongly) correlated. Thus, since pensions are liabilities to the fund while the longevity bond coupons are positive cash flows, the fund must go long on the longevity bond.

This strong intuition can be led further thanks to the algebraic demonstrations carried out in Menoncin (2006). The paper analyses the asset allocation problem for a representative consumer (with constant relative risk aversion) who invests in a stock (or a portfolio of stocks), an ordinary bond, a longevity bond and liquidity. The only two risks the consumer-investor must hedge against are the interest rate risk and the longevity risk. This setting is akin to those already developed in the financial literature (see, for instance, Boulier et al., 2001; and Battocchio and Menoncin, 2004) which nevertheless do not take into account the longevity bond.

The results we are going to present are valid if the interest rate and the force of mortality are instantaneously uncorrelated but not necessarily independent. In other words, the risk sources affecting the two stochastic variables are assumed to be different but interest rate and mortality are allowed to depend on each other.[3]

One of the main results in the asset allocation literature states that any asset is used for hedging against the risk it has the highest correlation with.As a consequence, a longevity bond should be the only asset used for hedging

Table 2: Values of parameters

Interest rate (r)		**Force of mortality (λ)**	
$dr=\alpha_r(\beta_r-r)dt+\sigma_r dW_r$		$d\lambda=-\frac{1}{b}(\phi-\lambda)\,dt+\sigma_\lambda dW_\lambda$	
Mean reversion, α_r	0.20	Mean reversion (diversion), b	10.5
Mean rate, β_r	0.05	Mean value, ϕ	0.001
Diffusion, σ_r	0.01	Diffusion, σ_λ	0.001
Initial value, r_0	0.05	Initial value, λ_o	0.00123
Risk premium, ξ_r	-0.46	Risk premium, ξ_λ	-0.006
Stock (S)		**Subjective parameters**	
$\frac{dS}{S}=(r+\sigma_S\xi_S+\sigma s_r\xi_r)dt+\sigma_{Sr}dW_r+\sigma_S dW_S$		Risk aversion, δ	3
Stock diffusion on interest risk, σ_{Sr}	-0.18	Subjective discount rate, ρ	0.05
Stock own diffusion, σ_S	0.18		
Risk premium, ξ_S	0.40		

against the longevity risk. Instead, the ordinary bond should be used for hedging against the interest rate risk while the stock shouldn't play any hedging role. In Menoncin (2006) these main intuitions are confirmed together with some other results that can be summarised as follows:

- The optimal portfolio contains a so-called speculative component which tends to disappear while the consumer's risk aversion increases (and becomes zero for an infinitely risk averse investor).
- The optimal portfolio contains as many hedging components as risk sources.
- An infinitely risk averse consumer invests only in ordinary bond and liquidity (see also Wacther, 2003). In this case the optimal weight of the ordinary bond coincides with the ratio between the duration of a perpetual annuity and the duration of the bond.
- The stock does not play any hedging role (it is just used for speculative purposes).
- The longevity bond is used for hedging only against the longevity risk.
- Let $\mathcal{D}_B$ be the duration of the ordinary bond and $\mathcal{D}_L$ the duration of the longevity bond. Then the optimal amount of wealth to be invested in the longevity bond is taken from the ordinary bond by the ratio $\frac{\mathcal{D}_L}{\mathcal{D}_B}$ and from liquidity by the ratio $1-\frac{\mathcal{D}_L}{\mathcal{D}_B}$ without changing the amount of stock. Let us highlight that such a result does not depend on the investor's risk aversion.
- The amount of money invested in ordinary bond, longevity bond, and liquidity, does not depend on the longevity risk but just on the other risks.
- If the durations of both the longevity and the ordinary bond coincide[4] then the amount of money to be invested in the longevity bond are taken only from the money invested in the ordinary bond (by changing neither the liquidity nor the stock).

It is worth stressing that these results hold for any risk aversion level and for both a complete and an incomplete market. Unfortunately, in the case of an incomplete market, we are not able to compute the exact value of the optimal portfolio weights. Accordingly, in order to carry out a numerical simulation, in the following section we build a market model where there are as many (linearly independent) assets as risk sources.

A numerical simulation

The results we have summarised in the previous sections are very general and characterise a wide range of different models for the financial market. In this section we present a particular case allowing to compute the exact weight that must be given to any financial asset. The financial market we take into account is formed by:

- One riskless asset whose return $r(t)$ is stochastic and follows a mean reverting process (Vasiček, 1977). The equation driving the interest rate and its parameters (consistently chosen with Babbs and Nowman, 1998, 1999) are shown in Table 2.
- One rolling (constant time to maturity: 25 years) bond whose market price of risk (ξ_r) is negative (see Table 2), since the bond is negatively correlated with the interest rate shocks.
- One stock (which could be thought of as a market index) whose price $S(t)$ is (negatively) correlated with the interest rate. The equation driving the asset price and its parameters (consistently chosen with Rudolf and Ziemba, 2004) are shown in Table 2.
- One rolling (constant time to maturity: 25 years) longevity bond whose market price of risk (ξ_λ) is negative (see Table 2) since the bond is negatively correlated with the force of mortality shocks.

The subjective discount rate for the consumer-investor is set equal to the equilibrium value of the riskless interest rate (0.05) while the constant risk aversion parameter is assumed to be $\delta = 3$. The force of mortality is assumed to follow a Gompertz-Makeham law with a stochastic error. The deterministic version of this law is:

$$\lambda(t)=\phi+\frac{1}{b}e^{\frac{t-m}{b}} \qquad (3)$$

where ϕ measures accidental deaths linked to non-age factors, while m and b are modal and scaling parameters of the distribution, respectively (the values in Table 2 and $m = 88.18$ are taken from Milevsky, 2001). Once the value of $\lambda(t)$ is differentiated with respect to time, we obtain an Ornstein-Uhlenbeck process as in Table 2 (which is not mean reverting). The initial value of the force of mortality (λ_0) is computed from Equation (3) for an agent aged 25 years ($t = 25$). We assume the force of mortality diffusion (σ_λ) to be one tenth of the interest rate diffusion.

Finally, the market price for the force of mortality risk (ξ_λ) is the most difficult parameter to estimate since there are no actively traded longevity bonds. We have arbitrarily given the longevity bond an excess return which is 2.5% higher than the excess return on the ordinary bond. We stress that an increase (reduction) in this value just leads to an increase (reduction) in the longevity bond optimal weight, without modifying its behaviour through time.

The results of the simulation carried out for a 60-year period are shown in Figures 1-3. On the horizontal axis, the time elapsed from the age of 25 is measured (accordingly, an abscissa of 30 corresponds to an age of 55). On the vertical axis the optimal asset weights are measured.

Some comments on the simulation shown in Figures 1-3 follow:

- Figure 1: the optimal weight of the longevity bond is decreasing through time since the need to hedge against the longevity risk becomes lower and lower. Since the volatility of λ is assumed to be constant and λ increases

3 We recall that in the macroeconomic literature agents optimally behave according to a dynamic equation linking the interest rate to the growth rate of the active population.
4 This happens if: (i) the force of mortality is independent of the asset returns, and (ii) the coupon structure of both the longevity and the ordinary bond are the same.

Figure 1. Optimal weight for longevity bond

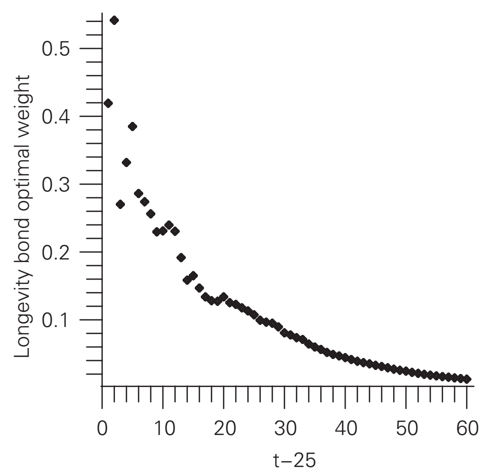
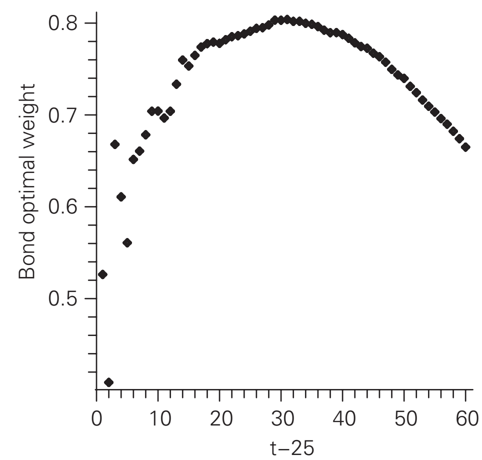


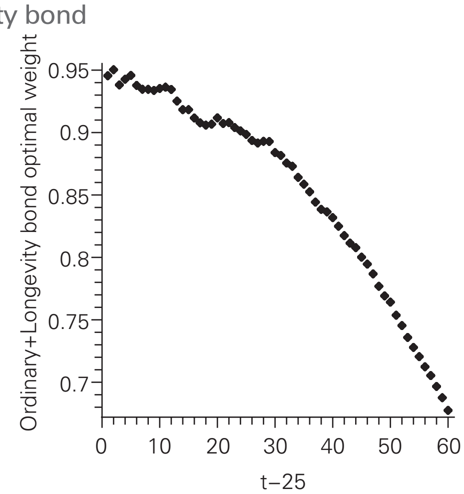
Figure 2. Optimal weight for ordinary bond



through time, then the relative weight of its volatility decreases. Accordingly, the longevity bond optimal weight is much more volatile during the first years.

- Figure 2: the optimal weight of the ordinary bond is highly volatile during the first years. This is due to the stochastic behaviour of the force of mortality λ. During the first years, the ordinary bond weight is negatively correlated with the longevity bond weight. The concave shape is due to the reducing magnitude of the longevity bond (which has a dominant weight during the first period).
- Figure 3: during the first years, the sum of both the ordinary and the longevity bond is much less volatile than the single bonds. This is due to the result that such a sum does not depend on the longevity risk but just on the interest rate risk (see point 7 in the previous section). This panel also shows what the optimal weight of an ordinary bond would be if there were not a longevity bond.
- The optimal weight of stock is constant since we have assumed constant market prices of risk and constant risk aversion too.
- In order to have an explicit solution for the optimal portfolio, short sales are allowed. In fact, the sum of all the risky assets in the optimal portfolio exceeds 100%. Accordingly, it is optimal to go short on the riskless asset (that is, borrow money at the riskless interest rate) as in other models where short sales are allowed (see, for instance, Boulier et al., 2001).

Figure 3. Optimal weight for both ordinary and longevity bond



Conclusion

During the last few years, some European countries have been trying to switch from a public pension system (mainly of the pay-as-you-go type) to a private pension system based on pension funds. In this context, policy-makers need to ensure that the new system is efficient since an early failure of participating institutions through misuse of funds is likely to cause a major setback to the whole reform program and to discourage older workers to join the new system.

The success or failure of a new pension system depends on its ability to use whatever capital has been amassed at the end of the accumulation phase to supply the pensioners with a reasonably sufficient regular income (during the distribution phase).

The importance of finding a suitable way to manage both phases is further supported by the figures provided by the United Nations population division (2002). In 2000, about 0.6 billion people (10% of the world's population) were over 60. By 2025 and 2050 this percentage will reach 15% and 21.1%, respectively. Almost half of the world's population is now in countries that are under the replacement rate fertility level (less than 2.1 children per woman). The life expectancy has been rising too. It is estimated that worldwide life expectancy at birth for men will rise from 61 in 1998, to 67 in 2025; and for women from 65 to 72 (80 in high income countries).

While the main risks borne during the accumulation phase (when workers pay contributions to the pension funds) can be managed with the usual strategies applied by investment funds, pension funds also have to face a new kind of risk: the so-called longevity risk. The recent attempt (by the EIB and BNP Paribas) to issue a so-called longevity bond paying a coupon which is proportional to the cumulative survival rate of a given population, testifies to the relevance of pension fund risk management for the financial community. We have shown in this paper that the new longevity bond would be a major step towards the creation of new financial-actuarial assets which are suitable for that particular kind of risk sharing.

Here, we have presented the intuition behind the main result, according to which a longevity bond, written on a population which should be as close as possible to the population subscribing the pension fund, must be held in a pension fund portfolio just reducing the amount of the ordinary bond and liquidity (proportionally to the durations of the two bonds). If both the longevity and the ordinary bond have the same duration, then the amount of wealth that must be invested in the longevity bond is entirely deduced from the ordinary bond. Accordingly, even if the optimal amount of money to invest in the longevity bond can be computed just after non-trivial estimations of the financial market variables, the management of the new asset does not seem to be too complicated. Once a pension fund has found the suitable amount of longevity bond to hold, it just has to take this money from the investment made in ordinary bonds and liquidity without altering the stocks in its portfolio. Finally, we have presented a numerical simulation which shows that the weight of a longevity bond in an optimal portfolio is definitely relevant. L&P

Francesco Menoncin, Dipartimento di Scienze Economiche, Università degli Studi di Brescia, Via S. Faustino,74/B, 25122 - Brescia, Italy. Tel: 0039-0302988806. Fax: 0039-0302988837. E-mail:menoncin@eco.unibs.it

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