Abstract: In May's *Expert Series*, *Les Clewlow* and *Chris Strickland* discussed the use of Monte Carlo simulation in energy risk management and introduced a series of models that they argued were suitable for the simulation of energy- and weather-dependent variables. Here, with *Michael Booth*, they apply these techniques to the modelling of electrical load

Spring loading

★ Volumetric risk is an important risk factor in many energy portfolios. Examples include the ability to price and calculate the risk of generation assets, demand-triggered caps, retail contracts, variable volume swaps, swing contracts and many others.

Load can be simulated directly by applying one of the models that we described in the May edition. However, it is well documented that weather – and in particular, temperature – has a major influence on the demand for power. Figure 1 plots the hourly temperature (in degrees Celsius), observed at Sydney Airport and system load data for the New South Wales (NSW) region of Australia for the period of calendar year 2002.

Figure 1 shows that in both the summer and winter periods system load increases – demand increases in winter when it is used for heating and again in summer because of the extensive use of air conditioning. The seasonal effects observed in this figure will be different to those in the northern hemisphere. Clearly the timing of the summer and winter seasons are reversed. Moreover, demand for power is also affected by alternative energy sources for heating – such as gas or oil – during winter. Note however, that these differences do not affect the analysis that follows. Figure 2 plots the same data sets but for a shorter time series, on a seven-day period in January 2002.

In figure 2 we observe a distinct intraday demand profile. Demand is low at night and higher during the day. Also, we can see that the different weekdays (the first five days in figure 2) have similar demand profiles to each other, but that they exhibit different load profiles to weekends (the last two days in figure 2). The main factors that influence electricity demand (at the aggregated regional level) are the day of the week, the time of day and temperature. Figure 3 plots the relationship between the NSW system load and Sydney Airport temperature for a particular trading period - 10am on Thursdays.

In figure 3 we see that, for a given time of day, the relationship between load and temperature can be approximated by a quadratic function (solid line), indicating increased power consumption at both high and low temperatures. The quadratic fit captures the overall relationship between load and temperature reasonably well. However, it is clear that other factors have a significant influence on the intra-day load pattern, giving considerable variation around the quadratic

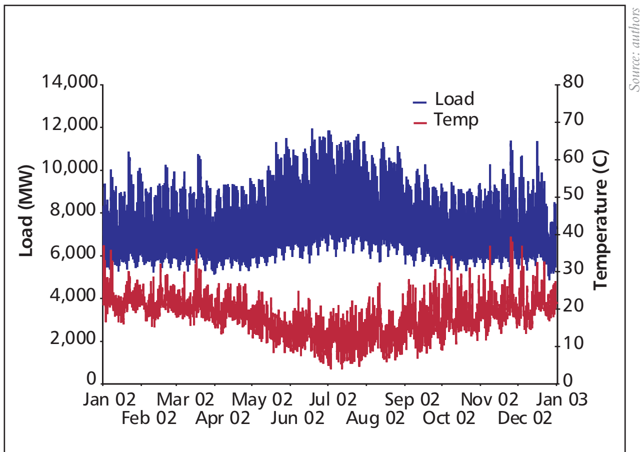

Fig 1. Temperature and system load for NSW (Jan – Dec 2002)

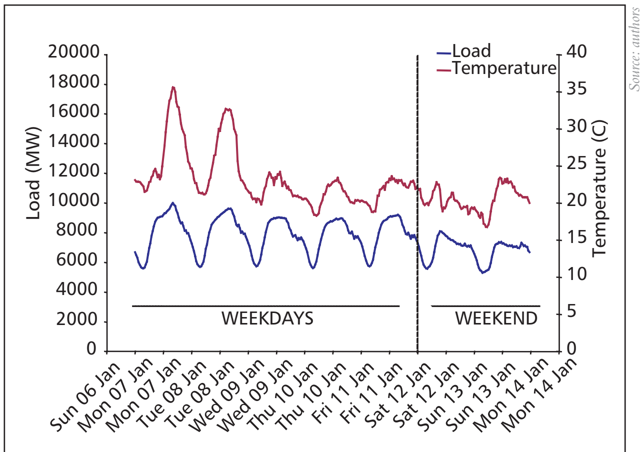

Fig 2. Temperature and load for New South Wales

temperature–load curve. In the NSW example, the response of load to temperature is symmetrical, with both relatively cold and hot conditions giving equal load increases. In many regions, electricity is not the main source for heating and so the load response for cooler temperature is much less than for higher temperatures, which in most regions leads to increased use of air conditioning and thus increased load. In these cases a bi-linear or quadratic fit can be used. Fitting more complex functions, such as higher order polynomials, tends not to work well because of the high variability of load at a given temperature.

One of the problems of naively fitting a quadratic form to the load-temperature data is that it implies that at very low or very high temperatures the load increases without limit. Clearly this is not the case, with physical system constraints limiting the maximum load. In practice, any simulated load should be capped by the maximum system load.

We conclude from figures 1 to 3 that load is affected strongly by the prevailing weather conditions and that temperature is a fundamental driver of the load dynamics. We will now make explicit this link between temperature and load dynamics by modelling system load as a transformed risk factor – we will simulate temperature and use the fitted quadratic relationship to obtain a simulated load.

The quadratic analysis of figure 3 can be repeated for each half-hourly period of our data. We find that the temperature–load relationship varies not only by half-hourly periods but exhibits noticeably different patterns for business and non-business days and across different seasons of the year. Figures 4 and 5 show the results of this analysis for the summer season temperature–load data in figure 1 for Thursdays and Sundays respectively.

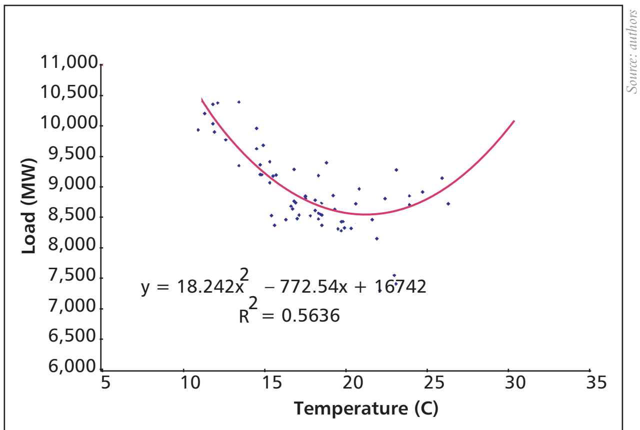

Fig 3. **NSW load and Sydney Airport temperature for the period ending 10am Thursday**

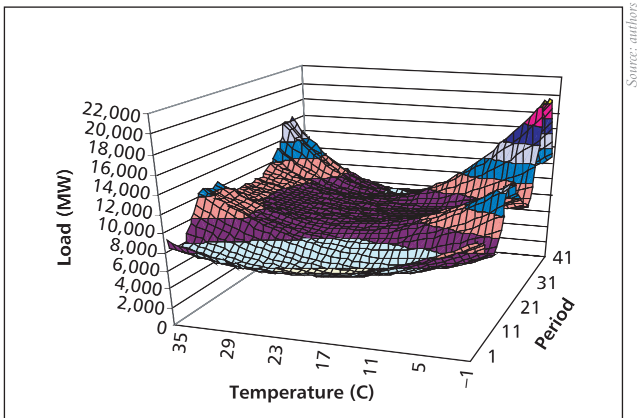

Fig 4. **NSW load versus Sydney Airport temperature model – Tuesdays**

Temperature–load model

As we noted earlier, the temperature at Sydney Airport does not completely explain the observed load in NSW. Apart from time of day and whether the day is a business or non-business day, the total load in a large region is affected by a wide range of factors, including sub-regional variations in temperature. For example, it may be cooler/warmer in other parts of NSW when compared to Sydney Airport; other weather variables – for instance, humidity, wind strength and direction; and the thermal properties of buildings – some buildings retain heat better than others. The net effect of all these other factors is essentially random – as demonstrated by the scatter of the points about the quadratic fit in figure 3. Rather than attempting to model these factors directly (and therefore making the load model more complex and difficult to use), we can combine the effects of these other factors into a random 'residual load' risk factor.

Given the quadratic assumption we have made for this article, the relationship between temperature and load is specified by the following relationship between the (total) load $\tilde{L}(t)$ and the temperature $H(t)$

$$\tilde{L}(t) = a(t) + c(t)\left(H(t) - H_{\min}\right)^2 + L(t) \qquad (1)$$

Here $a(t)$ represents the time-varying base load when the temperature is at H_{min}, the temperature at which it is typically observed that there is no preference to consume electricity for heating or for cooling. This temperature is usually assumed to be 18.3 °C (65 F). However, it is an adjustable parameter for modelling purposes. The quadratic response of load to the temperature difference from H_{min} is determined by $c(t)$. The residual load $L(t)$ encapsulates the 'residual' uncertainty of the (total) load once the temperature-dependent component has been removed. It is modelled as a normal version of the mean-reversion jump-diffusion (MRJD) process, represented by equation 2 below. Predictable patterns in the residual load resulting from the other sources of load variability mentioned earlier can be captured by a time-dependent mean-reversion level.

Temperature and load simulations

The procedure for simulating total load is as follows:

- Simulate temperature
- Using the quadratic relationship between load and temperature, transform the simulated temperature risk factor into the 'temperature-dependent' load
- Simulate residual load
- Add the residual load to the temperature-dependent load to produce the total load

In order to simulate the weather variable, we specify a normally distributed mean-reversion model for temperature. In addition, the user might want to allow for jumps – although this might not be intuitive, temperature levels can change significantly between one hourly observation and the next. The MRJD process for temperature can therefore be represented by an equation of the form

$$dH(t) = \alpha(\mu(t) - H(t))\,dt + \sigma(t)\,dz(t) + \kappa(t)\,dq(t) \qquad (2)$$

Here $H(t)$ is the temperature variable, α is the mean-reversion rate, $\kappa(t)$ is the proportional jump size, $dz(t)$ is the diffusion process and $dq(t)$is the jump process. Historical data can be used to specify the mean-reversion level $\mu(t)$ – or information from the expected weather pattern (weather forecast) can be incorporated. Time-dependent volatility levels $\sigma(t)$ can be used to take into account of the seasonal variances in temperature that we observed in our May article .

Figure 6 shows three joint simulations of the temperature–load model and compares them with the actual NSW system load from figure 2 over a one-week period in January 2002. The parameters of the temperature process in equation 1 were calibrated to historical summer data, as was the temperature–load model represented by figures 4 and 5 and the residual load process. It is straightforward to observe the effect that the simulated temperature has on load.

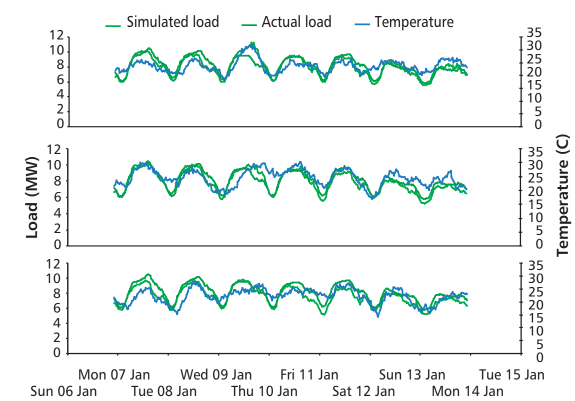

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Fig 6. Simulated and actual loads

Temperature–load parameter estimation

Figure 7 provides an overview of the joint temperature–load modelling estimation process. The inputs to the temperature–load model are: (i) historical temperature observations from a suitable weather station, (ii) historical load data and (iii) a historical holiday calendar. This data is used by the temperature–load model-fitting procedure to produce the quadratic fit coefficients for each period, day type, and season. The value of H_{min} is also calibrated if it has not been preset. The MRJD parameters (mean-reversion level, mean-reversion rate, volatility and jump parameters) for both residual load and temperature are then

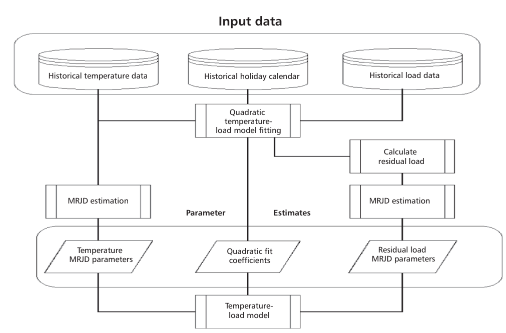

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Fig 7. Overview of joint temperature–load modelling estimation process

estimated. The above estimates may then be used in the simulation of temperature and load as outlined above.

This approach to modelling load allows the realistic simulation of load with a model that has an intuitive structure and parameters. In our next article, we will extend this type of 'causal' relationship modelling to the joint modelling of load and power price. ER

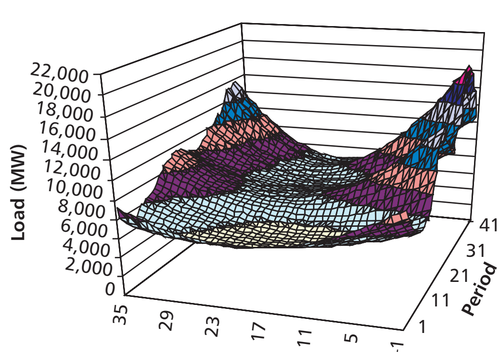

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Fig 5. NSW load versus Sydney Airport temperature model – Sundays

Les Clewlow and Chris Strickland are principals of Lacima Group, and authors of the forthcoming book *Energy Risk Management: Applications Using Simulation*. Michael Booth is a senior quantitative analyst with Lacima Group.
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