Wrong-way risk modelling

Beyond its potential impact on counterparty risk exposure, the wrong-way risk arising in some derivatives transactions raises important modelling challenges. Christian Redon presents two suitable models based on conditional expected exposure. Among straightforward applications, he focuses on country risk

If a counterparty to an over-the-counter derivative transaction defaults, the subsequent losses suffered by the other party are equal to the replacement cost of the OTC derivative. If the replacement cost is positive, the exposed counterparty suffers an economic loss, since it would have to pay this amount to maintain its financial profile unchanged after the default event has occurred. The replacement cost is by definition its mark-to-market value. Hence, the amount at risk in an OTC derivative – the exposure at default – is its mark-to-market value immediately after the date of default if it is positive, zero if it is negative. Moreover, the higher the mark-to-market value after default, the bigger the subsequent credit loss.

Broadly speaking, a wrong-way exposure can be understood as a derivatives transaction for which there is a significant positive dependency between the counterparty's default and the mark-to-market value of the transaction. Given the default of the counterparty, the replacement cost is on average more substantial than otherwise expected. Such possible combinations are met when the following adverse economic circumstances increase significantly and simultaneously:

- the probability of the counterparty defaulting conditional on the economic factors; and
- the probability of having a high mark-to-market value conditional on the economic factors.

Adverse economic circumstances can be either specific to a counterparty and a contract, or general in that they influence many counterparties because they are linked to general market risk factors. A classic example of specific wrong-way risk is the sale of a put option to a company on its own stock. If the company defaults, its share price will be extremely low and therefore the put option value will be very high. An example of general wrong-way risk would be entering into an interest rate swap where a counterparty pays variable and receives a fixed rate, but whose solvency is significantly dependent on interest rates. This could be the case, for instance, if it is heavily relying on borrowing short-term or at variable rates. In this case, an increase in interest rates will raise both the mark-to-market value of the transaction and the likelihood of the counterparty defaulting.

Basel II and wrong-way risk

The 90-page July trading book addendum to the revised Basel capital adequacy framework paves the way for an extensive use of expected positive exposure (EPE) – that is, the expected positive mark-to-market value averaged over time, as the exposure at default estimate of derivatives. As such, it will certainly foster EPE as a worldwide standard for measuring derivative counterparty risk among financial professionals. However, the use of EPE for regulatory capital calculations is subject to numerous critical requirements. One of them is wrong-way exposure.

According to point 194 of the trading book addendum, EPE should be adjusted to reflect wrong-way risk when it is significant. This issue is defined in points 141–142: "(141) Banks must be aware of exposures that give rise to a greater degree of general wrong-way risk. (142) A bank is said to be exposed to 'specific wrong-way risk' if future exposure to a specific counterparty is expected to be high when the counterparty's probability of default is also high. For example, a company writing put options on its own stock creates wrong-way exposures for the buyer that are specific to the counterparty. A bank must have procedures in place to identify, monitor and control cases of specific wrong-way risk, beginning at the inception of a trade and continuing through the life of the trade."

One answer to this issue is the use of Monte Carlo simulation, which, thanks to its flexibility, offers numerous ways of adjusting for wrong-way exposure by correlating default and derivatives exposure. However, they offer poor insight into how this correlation influences exposure. Moreover, wrong-way exposure is seldom considered in modelling literature, which probably reflects the lack of explicit modelling techniques regarding this important matter. Two significant attempts to model wrong-way risk have been made (Levy, 1999, and Finger, 2000). Since then much knowledge of derivatives counterparty risk has emerged, notably through EPE measures and their regulatory use, the substance of which can be found in Canabarro, Picoult & Wilde (2003) and the Basel Committee on Banking Supervision trading book addendum (2005). In comparison, little progress seems to have been made with regard to wrong-way risk modelling.

As a further contribution to this issue, we suggest some techniques to account for wrong-way risk in EPE modelling. All these techniques are analytical and thus easily transposable to Monte Carlo simulation. Most notably we adjust the definition of EPE to reflect wrong-way risk and suggest practical implementations in what is probably the most frequently encountered situation of wrong-way risk: country crises.

How are country risk and wrong-way risk related? The classic example is where an institution has a short position in a currency dealt with a counterparty based in this country – for example, selling bahts to a Thai counterparty. One can reasonably assume that if a major country crisis leads to a counterparty defaulting, the currency will have severely depreciated. The value of the derivative will then have jumped to what would have been seen before this event as an unexpectedly high exposure, leading the institution to experience a potentially severe loss. Conversely, if an institution is buying the currency at risk, it will naturally be protected against any country risk, since if a crisis occurs, the value of the derivative will generally be negative. The consequence for the institution will be to experience no loss. In this case, the exposure is referred to as right-way exposure.

Of course, standard measures such as 'pure' expected positive exposure or peak exposure are irrelevant for evaluating such events, unless these already include them. Given this perspective, we focus on a definition of exposure that is conditional on the counterparty defaulting and then tailor models that allow for calculating it in different ways.

The rest of the article is as follows. We first recall the notion of expected exposure and introduce the important concept of conditional expected exposure. We then introduce a first model of conditional exposure measurement in the context of country risk and illustrate it with some numerical examples. The last section of the article is dedicated to a general model that captures a wider range of dependencies and enables us to calculate exposure by a simple change of probability.

Conditional exposure

Let us consider a derivatives transaction for which we write:

- *MtM*(*t*), the mark-to-market value at a date *t*. If *t* is higher than the maturity date of the transaction, the *MtM*(*t*) value will be nil;
- τ, the default date of the counterparty, and *q* its probability density function; and
- ρ(*t*), the discount factor between today and *t*.

Given these notations, expected exposure at a given date *t* can be expressed as follows:

$$e(t) = E\left[\left(MtM(\tau)\right)_+ \middle| \tau = t\right] \tag{1}$$

where:

$$(x)_+ = \begin{cases} x & \text{if } x > 0 \\ 0 & \text{else} \end{cases}$$

and *e*(*t*) is the expected exposure conditional on the counterparty defaulting. Put more intuitively, it is the average amount that will be owed by the counterparty should it default on date *t*.

Note that this notion is identical to the standard expected exposure in the particular case where the default process τ and the mark-to-market value process *MtM*(*t*) are independent. The latest is generally the base case assumption and yields the 'usual' definition of expected exposure:

$$\bar{e}(t) = E\left[\left(MtM(t)\right)_+\right] \tag{2}$$

By expected exposure, we will always refer to the conditional approach, so as to have a general framework that allows for dependencies between exposure and default, such as wrong ways and right ways. In a general framework, expected loss is written as:

$$\begin{aligned} EL &= E\left[\rho(\tau)\, LGD \left(MtM(\tau)\right)_+\right] \\ &= E\left[\rho(\tau)\, LGD\, E\left[\left(MtM(\tau)\right)_+ \middle| \tau\right]\right] \\ &= E\left[\rho(\tau)\, LGD\, e(\tau)\right] \end{aligned} \tag{3}$$

where *LGD* is the loss-given default and *e*(*t*) is the conditional expected exposure:

$$e(t) = E\left[\left(MtM(t)\right)_+ \middle| \tau = t\right] \neq E\left[\left(MtM(t)\right)_+\right]$$

When expected loss is calculated under a risk-neutral probability, it can be interpreted as the theoretical market price for hedging the credit loss. A hedging strategy can be shown to exist, and the theoretical cost of hedging is then given by expected loss calculated using credit spreads. On the other hand, when expected loss is calculated under historical probability, it represents the anticipated average loss.

Another way to look at it is to condition the expectation on stressful events. Canabarro, Picoult & Wilde (2003) use unconditional EPE with the Basel II asymptotic single-risk-factor credit model, in which economic capital is asymptotically defined as an expectation conditional on downturns in the credit cycle, the latest being represented by the single factor. Conditional EPE could also reflect correlation between the single credit factor and exposures. Such dependencies are referred to in the revised bank capital adequacy framework as general wrong-way exposures or EPE conditional on the 'bad state' of the economy.

Country risk

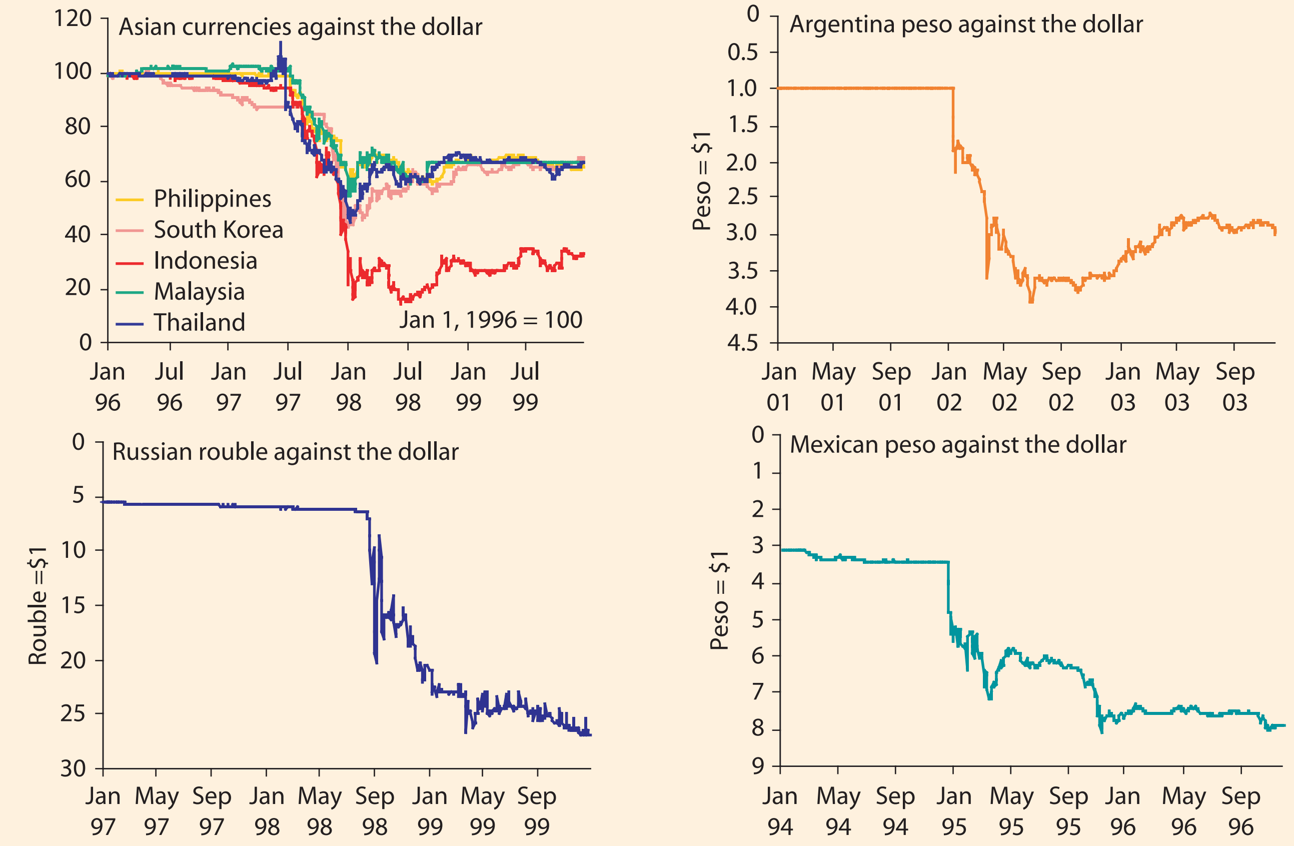
A widely accepted definition of country risk is given by the Basel Committee (1982): "Country risk is taken to refer to the possibility that sovereign borrowers of a particular country may be unable or unwilling, and [the] other borrowers unable, to fulfil their foreign obligations for reasons beyond the usual risks that arise in relation to all lending."

Many factors may produce such an outcome, thus making country risk a difficult concept to define with precision: "The risks range from the consequences of official actions or important sociopolitical changes in the borrowing country to largely unpredictable events such as natural disasters or external shocks arising from global phenomena like world depression or the consequences of an oil price rise."

Hence the assessment of country risk is often deemed "an art in which a significant degree of unpredictability must be acknowledged". Despite this general difficulty, recent history tells us that country crises tend to follow a somewhat predictable pattern:

- a significant percentage of counterparties based in the country default, together with a sharp downturn in the economy; and
- there is a strong depreciation of the country currency.

1. Currency crises in the past decade



A few examples over the past decade include the Mexican ‘tequila crisis’ in 1994, the fall of the Asian tigers in 1997, the Russian crisis in 1998 and the more recent Argentina default in 2001–2002. Figure 1 shows how these crises have translated into the devaluation of exchange rates, with a dip of 40–90% of the currency’s value generally occurring within the year following the start of the crisis. For this article, we will stick to this descriptive version of country crises. For more on the explanatory factors of country risk, one can refer to Borio & Packer (2004).

To give some insight into the impact a country crisis might have on a derivative’s counterparty risk, let us now focus on typical transactions exposed to wrong-way risk. Consider a currency swap where the bank pays floating rate in a risky currency and receives floating rate in a strong currency, with an exchange at maturity of principal N, here expressed in strong currency. For more background on currency swaps and their valuation, one can refer to Hull (2005). Assume also that the counterparty paying the strong currency at maturity is based in the country of issue of the risky currency. Similar transactions bearing the same kind of risk typically include:

- FX forwards in which the bank delivers the risky currency; and
- put options bought on the risky currency.

If the currency swap is entered into today at the prevailing market conditions, an approximate[1] formula for its mark-to-market value at a given future date t is:

$$MtM(t) = \left(1 - \frac{S_t}{S_0}\right) N \qquad (4)$$

where S_t is the exchange rate prevailing at time t. Note that $MtM(t)$ is a decreasing function of S_t, so the lower the exchange rate, the higher the mark-to-market value. Consequently, a risky currency depreciation will automatically trigger an increase in the exposure.

Given the recurring features of the country crisis, we will assume that should a country crisis occur at a time τ_P, the following will happen:

- a given counterparty based in country P will have a probability p of defaulting, so writing τ for the time of default we will thus have:

$$P\left(\tau = t \middle| \tau_P = t\right) = p \qquad (5)$$

- and the currency will depreciate on average by a $d\%$ factor:

$$E\left[\left(MtM\left(\tau_P\right)\right)_+\right] = d\%N \qquad (6)$$

The conditional probability of default reflects the impact of the crises on the counterparty solvency. For instance, if $p = 1$, we will have the implication $\tau_p = t \Rightarrow \tau = t$ for all t. Given that all counterparties do not necessarily default on occurrence of a crisis, we should instead have $p \le 1$. Note that p can be interpreted either as a counterparty-specific parameter or as a general proportion of counterparties defaulting given a country crisis.

The estimation of the $d\%$ of the second assumption is also an issue. Having a quick look at the recent currency crisis as shown in figure 2, one can imagine that every value above 50% could be plausible. In the absence

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[1] *This is a formula with assumptions such as continuous floating-rate payments and an arbitrage-free set-up*

of more relevant information, an estimation of *d*% over a period of one year following the start of the crisis seems to be a reasonably conservative assumption, since a default and a loss are triggered by an early termination event. Such events are likely to happen on missed coupon payments and even more likely on principal payments. Therefore, anything from three months to a year following the crisis is acceptable as a termination date. As a consequence, a 50–70% value for *d*% would seem to fit the observed average depreciation in a slightly conservative manner.

We now estimate conditional expected exposure using the following general calculation, which holds when the country and the counterparty have strictly positive densities of default:

$$\begin{aligned} e(t) &= E\left[\left(MtM(\tau)\right)_+ \middle| \tau = t\right] \\ &= \lim_{h\to 0} \frac{E\left[\left(MtM(\tau)\right)_{+\{t<\tau\le t+h\}}\right]}{P(t<\tau\le t+h)} \\ &= \lim_{h\to 0} \frac{\begin{array}{c} E\Big[\left(MtM(\tau)\right)_+ 1_{\{t<\tau\le t+h\}\{t<\tau_P\le t+h\}} \\ +\left(MtM(\tau)\right)_+ 1_{\{t<\tau\le t+h\}\{\tau_P\le t,\tau_P>t+h\}}\Big] \end{array}}{P(t<\tau\le t+h)} \\ &= \lambda_t E\left[\left(MtM(t)\right)_+ \middle| \tau=t,\tau_P=t\right] \\ &\quad +(1-\lambda_t)E\left[\left(MtM(t)\right)_+ \middle| \tau=t,\tau_P\ne t\right] \end{aligned} \tag{7}$$

with:

$$\lambda_t = P(\tau_P = t | \tau = t) = \frac{q_P(t)}{q(t)} p \tag{8}$$

where $q(t)$ and $q_P(t)$ are the probability density functions of τ and τ_P taken at time t.

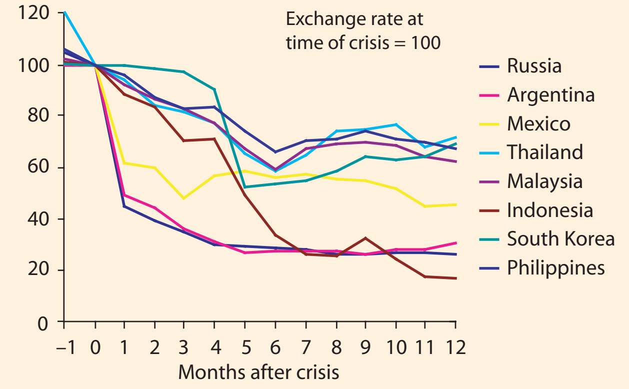
We therefore see that expected exposure can be estimated as a simple weighted average of:

- the expected exposure conditional on a severe country crisis; and
- the expected exposure conditional on no country crisis occurring.

Expected exposure conditional on a country crisis not occurring can be reasonably approximated by the unconditional expected exposure, which holds for countries that are not on the brink of a crisis and is a conservative assumption for others. Using this assumption, we obtain the following general formula:

$$e(t) = \lambda_t (d\%N) + (1-\lambda_t)E\left[\left(MtM(t)\right)_+\right] \tag{9}$$

2. The common pattern of exchange rates post-crisis



We see that expected exposure can be simply deduced from the unconditional expected exposure and a λ_t weight. The latter represents the proportion of the crisis among every possible situation where the counterparty defaults. Given an assumed value of p and depending on the application and available data, its estimation can be based on:

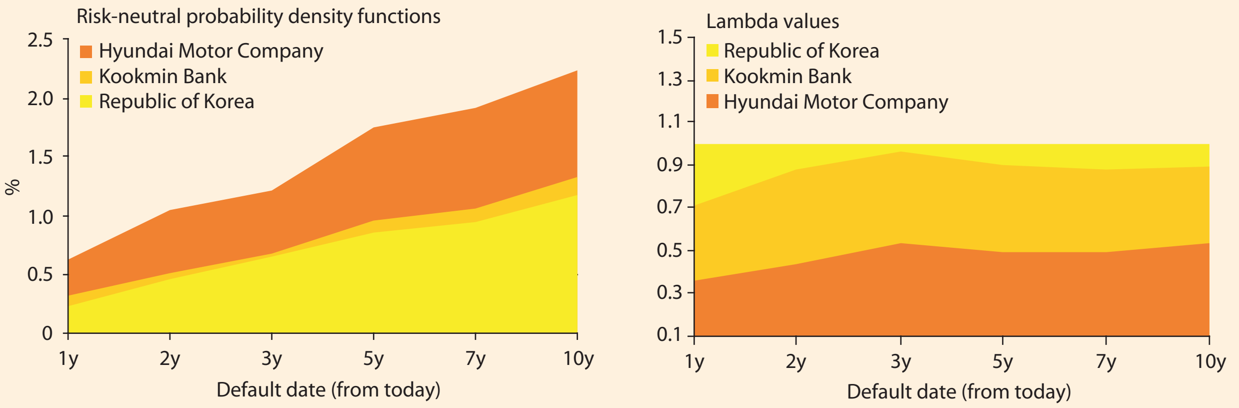
- the historical default probabilities that match given ratings of the counterparty and the country; and
- credit default swap (CDS) implied probabilities of default. For an explanation of how to derive risk-neutral probabilities of default from CDS prices, one can refer to Schönbucher (2003).

Figure 3 shows lambda values inferred from CDS prices for three South Korean counterparties. It is assumed in this example that $p = 1$, set to this conservative value for illustrative purposes. We use the Republic of Korea probability density of default as a proxy for q_P and compare it with the densities of default of two counterparties, Kookmin Bank and Hyundai Motor Company.

Under the previously stated assumption that the sovereign default triggers the counterparties' default, Kookmin Bank's lambda value, close to one after one year, shows that in the long run the bank and the sovereign default are closely related. This result seems quite reasonable, given that Kookmin is the largest South Korean bank and that major bank defaults are often the corollary of sovereign defaults.

On the other hand, Hyundai's lambda is around 50%. Under the assumption that a sovereign default triggers Hyundai's (that is, $p = 1$)

3. Lambda values for South Korean counterparties



this lambda value can be interpreted as the financial market seeing its default risk split into approximately equal shares of pure corporate risk and country risk.

A general model of wrong-way exposure

A wrong-way exposure is a derivatives transaction for which there is a significant positive dependency between the counterparty's default and the mark-to-market value. The country risk model presented above relates to a general wrong-way risk, since the underlying country crisis has a potential impact on all the counterparties based in the country. Another example of what could be considered a general wrong-way risk is entering a forward contract in which oil is bought by a counterparty that could suffer significantly from rising oil prices, such as an airline. By comparison, specific wrong-way risk relates to such transactions where the relation between the counterparty default and the mark-to-market value is purely specific to the counterparty – for example, when:

- buying credit protection on a subsidiary of its parent company through a CDS; or
- buying a put option on a counterparty on its own stock.

Along with the general or specific issue, the type of dependency can range from very weak to very strong. In the previous section, we envisaged perfect 'tail' dependencies, where the default of the counterparty is related perfectly to a dramatic rise in the exposure. The above specific wrong-way examples can be plugged into this model very simply. Just replace the time of default of the country by the time of default of the counterparty or the parent company, and you will get the same binary description where conditional exposure is a convex combination of two simpler default/non-default conditional exposures:

$$\begin{aligned} e(t) = {} & \lambda_t E\left[\left(MtM\,(t)\right)_+ \middle| \tau = t, \tau_P = t\right] \\ & + (1-\lambda_t) E\left[\left(MtM\,(t)\right)_+ \middle| \tau = t, \tau_P \neq t\right] \end{aligned}$$

However, the airline example reflects weaker types of dependency that cannot be precisely addressed with such a binary model. When significant, such dependency can create as much damage as the binary one. The difficulty when addressing such dependency is to quantify its exact degree and nature. While statistics and probabilities provide numerous ways to model dependency, the most tractable in finance is without any doubt the multivariate Gaussian dependency. In particular, it has proved to be very successful in at least two major areas of financial risk:

- market risk (dynamic pricing models essentially rely on Gaussian dependencies, as do parametric or Monte Carlo value-at-risk models (see, for example, Mina & Yi Xiao, 2001, for an overview)); and
- credit risk (portfolio models also often rely on Gaussian dependencies – for instance, vendors' models such as CreditMetrics and KMV portfolio manager use a Merton's framework such as Gaussian latent variables to model credit dependencies among obligors (see, for example, Gupton, Finger & Bhatia, 1997)).

Gaussian dependency therefore seems a natural candidate to assess the intersection of what can be found in counterparty risk and wrong-way risk. An obvious model can be built following these steps:

- modelling defaults in a Merton-like framework;
- modelling exposure with an underlying Gaussian diffusion process; and
- calculating conditional exposure by taking into account a correlation between the two processes.

Finger (2000) used a comparable approach to build a single-period framework. We detail each step in a continuous time framework.

Step 1: a model for defaults. We present here a simplified version of the Merton model developed in Finkelstein *et al* (2003) where the default barrier is deterministic. In this firm-value model, the counterparty defaults when the value V_t of its assets crosses a barrier R that can be interpreted as its debt:

$$\tau = t \Leftrightarrow V_t = R \text{ and } V_s > R \text{ for } t < \tau \qquad (10)$$

The asset value is assumed to follow a lognormal process. Writing W_t for a standard Brownian motion, σ for the asset value volatility and μ its drift, we have:

$$V_t = V_0 \exp\left(\sigma W_t + \left(\mu - \frac{\sigma^2}{2}\right)t\right) \qquad (11)$$

The default probability can then be calculated as:

$$\begin{aligned} P(\tau \le t) &= P\left(\inf_{s\in[0,t]} V_s \le R\right) \\ &= P\left(\inf_{s\in[0,t]}\left[\sigma W_t + \left(\mu - \frac{\sigma^2}{2}\right)t\right] \le -\ln\left(\frac{V_0}{R}\right)\right) \\ &= N\left(-DD(t)\right) + e^{-2\lambda\theta} N\left(-DD(t) + 2\theta\sqrt{t}\right) \end{aligned} \qquad (12)$$

where $N(\cdot)$ is the standardised Gaussian conditional density function and[2]:

$$\lambda = \frac{\ln\left(\frac{V_0}{R}\right)}{\sigma} \quad \theta = \frac{\left(\mu - \frac{\sigma^2}{2}\right)}{\sigma} \quad DD(t) = \frac{\lambda}{\sqrt{t}} + \theta\sqrt{t}$$

Here, $DD(t)$ represents the distance to default, which is the relative distance between the firm's asset value and its default threshold, expressed as a number of standard deviations of asset value between now and time t. It can be interpreted as the credit quality of the counterparty, as the probability of default is a strictly decreasing function of this quantity. For more background on this important concept, see, for example, Crosbie & Bohn (2003).

In the current set-up, the distance to default is a function of λ and θ, where λ can be interpreted as the fraction representing current leverage, and θ the fraction representing the dynamic trend.

Step 2: modelling the exposure with a Gaussian model. We assume that the mark-to-market value of a transaction can be expressed as a function of a Brownian motion representing the underlying diffusion process:

$$MtM\,(t) = v(\mu_t + \sigma_t X_t) \qquad (13)$$

where X_t is a standard Brownian motion, μ_t is the drift of the underlying price process, σ_t is the volatility of the underlying price process and v is a real function.

In this framework, conditional exposure is written as:

$$e(t) = E\left[v(\mu_t + \sigma_t X_t) \middle| \tau = t\right]$$

We also define h as follows, when τ and W_t are independent:

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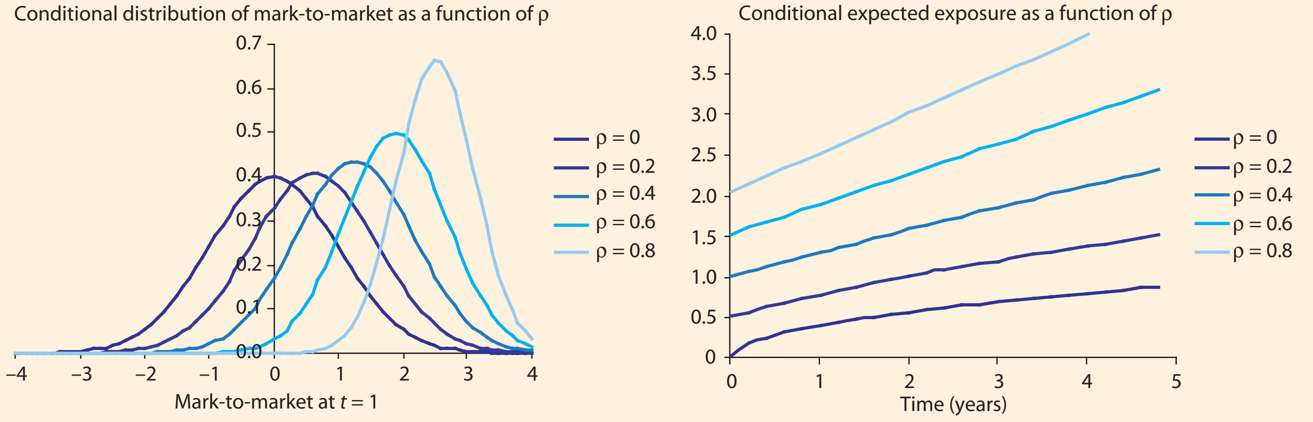
[2] *We use the properties of the first hitting time of Brownian motion (see, for example, Musiela & Rutkowski, 1998). When $Y_t = at + bW_t$ with constant a and b, we have:*

$$P\left(\inf_{s\in[0,t]}(Y_s) \le y\right) = e^{2ay/b^2} N\left(\frac{y+at}{b\sqrt{t}}\right) + N\left(\frac{y-at}{b\sqrt{t}}\right)$$

Here $a = \mu - (\sigma^2/2)$, $b = \sigma$ and $y = -\ln(V_0/R)$. We then simply write $\lambda = -y/b$ and $\theta = a/b$

4. Conditional distribution and exposure for different correlation values



$$\begin{aligned} e(t) &= E\left[\left(MtM(t)\right)_+\right] \\ &= h(\mu_t,\sigma_t) \end{aligned} \tag{14}$$

Step 3: calculating exposure with a wrong-way risk. Let us now go back to the general case with τ that follows the firm value model of the previous section. Writing ρ for the correlation between the default process W_t and the mark-to-market value process X_t, we have:

$$\begin{aligned} e(t) &= E\left[\left(MtM(t)\right)_+ \middle| \tau = t\right] \\ &= E\left[\left(v(\mu_t + \sigma_t X_t)\right)_+ \middle| \tau = t\right] \\ &= E\left[\left(v\left(\mu_t + \sigma_t\left(\rho W_t + \sqrt{1-\rho^2}\,\varepsilon_t\right)\right)\right)_+ \middle| \tau = t\right] \end{aligned} \tag{15}$$

where ε_t is a Brownian motion independent from τ and W_t. We also write $(\mathcal{F}_t)_{t>0}$ for the natural filtration of W_t. Knowing that $\tau = t \Rightarrow V_t = R \Rightarrow W_t = -\sqrt{t}\,DD(t)$, we find that:

$$\begin{aligned} e(t) &= E\left[E\left[\left(v\left(\mu_t + \sigma_t\left(\rho W_t + \sqrt{1-\rho^2}\,\varepsilon_t\right)\right)\right)_+ \middle| \mathcal{F}_t\right] \middle| \tau = t\right] \\ &= E\left[\left(v\left(\mu_t + \sigma_t\left(-\rho\sqrt{t}\,DD(t) + \sqrt{1-\rho^2}\,\varepsilon_t\right)\right)\right)_+ \middle| \tau = t\right] \\ &= h\left(\mu_t - \sigma_t\rho\sqrt{t}\,DD(t), \sqrt{1-\rho^2}\,\sigma_t\right) \end{aligned} \tag{16}$$

We therefore have:

$$e(t) = h(\bar{\mu}_t, \bar{\sigma}_t) \tag{17}$$

with:

$$\bar{\mu}_t = \mu_t - \sigma_t\rho\sqrt{t}\,DD(t) \qquad \bar{\sigma}_t = \sqrt{1-\rho^2}\,\sigma_t \tag{18}$$

The conditional exposure including wrong-way risk can therefore be measured as the unconditional exposure where the original drift and volatility would have been adjusted to reflect market prices conditional on default. This result is equivalent to a change of probability. Following the same arguments as above, it is easy to see that:

$$\begin{aligned} P\left(MtM(t) \le x \middle| \tau = t\right) &= P\left(v(\mu_t + \sigma_t X_t) \le x \middle| \tau = t\right) \\ &= P\left(v(\bar{\mu}_t + \bar{\sigma}_t X_t) \le x\right) \end{aligned} \tag{19}$$

The mark-to-market value distribution conditional on default is therefore equal to the unconditional distribution with adjusted drift and volatility. Moreover, only two parameters define the change of probability:

- the correlation ρ between the underlying default and mark-to-market value processes, which can represent both a general and a specific wrong-way risk, depending on the underlying market risk factor; and
- the distance to default $DD(t)$, which reflects the counterparty's credit quality.

A simple example. We set $\lambda = 2.54$ and $\theta = 0.61$, which yield a default profile representative of a typical BBB rated counterparty, with one-year and five-year probabilities of default respectively equal to 0.20% and 3.30%, in line with the rating agencies' cumulative default rates for this rating class. For illustrative purposes, we consider the simplest linear model with a short position in the risk factor X_t:

$$\begin{aligned} MtM(t) &= -(\mu_t + \sigma_t X_t) \\ h(\mu_t,\sigma_t) &= \sigma_t\sqrt{t}\,\phi\left(\frac{-\mu_t}{\sigma_t\sqrt{t}}\right) - \mu_t N\left(\frac{-\mu_t}{\sigma_t\sqrt{t}}\right) \end{aligned} \tag{20}$$

where $\phi(\cdot)$ is the standardised Gaussian probability density function. Figure 4 represents the conditional distribution and exposure for different correlation values and $\mu_t = 0$ and $\sigma_t = 1$ – that is, a zero average square root of time derivatives contract. In this example, the conditional exposure is written as:

$$\begin{aligned} e(t) &= \sqrt{1-\rho^2}\sqrt{t}\,\phi\left(\frac{\rho}{\sqrt{1-\rho^2}}DD(t)\right) \\ &\quad + \rho\sqrt{t}\,DD(t)\,N\left(\frac{\rho}{\sqrt{1-\rho^2}}DD(t)\right) \end{aligned}$$

As illustrated in figure 4, even low values of ρ lead to a substantial impact on the conditional expected exposure. For instance, this more than doubles in the short term with a correlation equal to 20%.

Conclusion

We have introduced the notion of conditional expected exposure, which enables us to account properly for potential dependencies between default and exposures when implementing EPE. We have also presented two distinct analytical models capable of reflecting wrong-way exposures in the context of conditional exposure. The first is more adapted to the frequent wrong-way situations occurring in country crises. It provides

REFERENCES

Basel Committee on Banking Supervision, 1982
Management of banks' international lending (country risk analysis and country exposure measurement and control)
At www.bis.org/publ/bcbsc122.pdf

Basel Committee on Banking Supervision, 2005
The application of Basel II to trading activities and the treatment of double default effects
At www.bis.org/publ/bcbs116.htm

Borio C and F Packer, 2004
Asset allocation: combining investor views with market equilibrium
Goldman Sachs Fixed Income Research

Clemen R and R Winkler, 1997
Assessing new perspectives on country risk
BIS Quarterly Review, December, pages 47–65

Canabarro E, E Picoult and T Wilde, 2003
Analysing counterparty risk
Risk September, pages 117–122

Crosbie P and J Bohn, 2003
Modeling default risk – modeling methodology
Moody's KMV, available at www.moodyskmv.com/research/whitepaper/ModelingDefaultRisk.pdf

Finger C, 2000
Toward a better estimation of wrong way exposure
RiskMetrics Journal, May, pages 25–40

Finkelstein V, J-P Lardy, G Pan, T Ta and J Tierney, 2003
Creditgrades – Technical Document
RiskMetrics Group

Gupton G, C Finger and M Bhatia, 1997
Creditmetrics – Technical Document
Morgan Guaranty Trust

Hull J, 2005
Options, futures and other derivative securities
Sixth edition, Prentice Hall

Levy A, 1999
Wrong way exposure
Risk July 1999, pages 52–55

Mina G and J Yi Xiao, 2001
Return to RiskMetrics: the evolution of a standard
RiskMetrics Group

Musiela M and M Rutkowski, 1998
Martingale methods in financial modeling
Second edition, Springer-Verlag

Schönbucher P, 2003
Credit derivatives pricing models
Wiley

a binary description of country crises in which conditional exposure is a weighted average of 'usual' expected exposure and the potential exposure that would occur on a crisis. The second model allows for wider types of dependencies in a Gaussian framework. A fundamental result is that diffusion parameters of market values can be adjusted to reflect this dependency. Potential applications of these analytical results include not only the evaluation of EPE on specific transactions but also possible adjustment of Monte Carlo procedures to reflect properly wrong-way risk in EPE implementation. ●

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