The Holy Grail of Optimality


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In this article the authors show that, besides portfolio optimisation driven by the economic equilibrium theory, no other risk adjusted performance measure leads to any substantial global advantage

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FINANCIAL RESEARCHERS, as well as financial institutions, devote a significant amount of resources on portfolio optimisation within capital markets. Given a fixed amount of cash, they all search for the optimal investment plan. Since all feasible portfolios have the same price, ie they can be bought and sold for the initial amount, it is not obvious whether there exists any superior portfolio composition. In an arbitrage free, respectively efficient market, two investments with the same market price are, first of all, equally valuable and, buying or selling them at the market price does not create any value. Many performance measures have been suggested to discriminate the good investments from the bad, to decide if a delegated manager is able to discriminate them. In our opinion, it is neither possible to measure economic value creation – due to skilled asset management with the commonly proposed performance measures – nor is one of these measures a good target for individual value optimisation.

Of course, this does not mean that all possible investment strategies are equally reasonable. Some companies will prefer risky assets, others rely on guaranteed liquidity. Every investor has her own preferences and her own objectives. This paper emphasises the importance of clear

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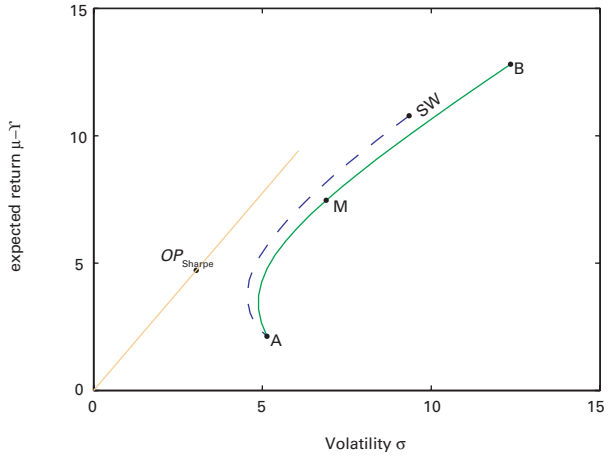
Fig 1. Classical application of the Sharpe ratio



objectives as a preliminary for any optimisation procedure. There is no sense in optimising a portfolio according to an unmotivated mathematical measure. This article claims that neither Sharpe-Ratio nor RoRaC or any other risk adjusted performance measure leads to any form of optimality other than optimising a mathematical formula.

Our argumentation is underpinned by the economic equilibrium theory, which suggests that all market participants agree to the prevailing market prices. Once new information emerges, the prices will change until the market is cleared and a new equilibrium is reached. It follows from that that the resulting market portfolio composition is the only possible optimum to which everyone would agree. Besides that, no other global optimal portfolio is reasonable, and therefore, any optimality depends on the specific preferences of the investors, for instance, those driven by their business model.

Sharpe ratio

The Sharpe ratio is one of the first performance measures that were proposed to the community of investors and it is still used as a preferred method for the evaluation of stock portfolios and funds (Sha66). It is defined as the ratio between expected excess profit (defined as the difference between expected profit μ and the risk-less interest rate r) and the risk associated with the investment. The standard deviation of profits is used as a numeric value for risk. Due to its simplicity, the Sharpe ratio is one of the most commonly applied methods for the evaluation of risky investments.

$$\text{Sharpe ratio} = \frac{\mu - r}{\sigma} \quad (1)$$

A portfolio optimisation based on Sharpe ratio has been successfully performed in the context of buy&hold stock portfolios. However, as soon as dynamically rebalanced portfolios are considered, the Sharpe ratio fails miserably. It has been pointed out various times that Sharpe ratios can be manipulated and optimised with dynamic trading (Der04, GW03).

Option portfolio OP_{Sharpe}		
Underlying	M	M
Type	Call	Call
Strike	0.4	1.005
Amount	1.8	-1.7

The first figure illustrates the classical application of the Sharpe ratio together with some of its exploits. The μ/σ-plot compares expected returns and standard deviation as a measure for risk. The points *A* and *B* refer to plain stocks with their mean on the y-axis and their volatility on the x-axis. The correlation in this example is 10%. A direct and well-received result is the obvious superiority of mixed portfolios, as indicated by the continuous line, over individual stocks. The market portfolio *M* consists of 50% stock *A* and 50% stock *B* and has the highest Sharpe ratio within the class of static portfolios. Next we consider the stop-win strategy (*SW*) that starts with a portfolio containing *B* and switches to the less risky stock *A* as soon as a certain profit threshold for the respective period is reached. This strategy is easily implemented by fund managers to display higher Sharpe-ratios. The stroked line connecting *SW* and *A* indicates portfolios that can be reached by mixing the two strategies. The Sharpe ratio can be further optimised with the option portfolio OP_{Sharpe}. From this portfolio composition we can again recognise the stop-win type property that arises from the short call. The line through OP_{Sharpe} – the option portfolio – indicates portfolios that mix options with risk-less investments.

The strategy with optimal Sharpe ratio was shown to produce a profit that is distributed according to a mirrored log-normal probability distribution (GW03). This strategy exhibits some similarities to a doubling strategy in roulette. The possible loss is unlimited but tiny profits are very likely. Under realistic conditions the Sharpe ratio can be easily manipulated by 20-30%. Consequently, this ratio can only serve as a rough performance estimate.

RoRaC

As we have seen in the previous section, a symmetric risk measure, like the volatility, weighs unexpected profits as risks. We can increase our Sharpe ratio with a stop-win strategy, since the reduced risk outweighs the reduced performance. Remedy was searched in downside risk measures that focused only on the risks arising from losses. A typical downside risk measure is the value at risk (VaR). Slightly less common is the expected shortfall, also known as conditional value at risk (CVaR), even though it has some appealing qualities. Under both risk measures stop-loss strategies and minimum guarantees are preferred.

Option portfolio OP_{RoRaC}	
Underlying	M
Type	Call
Strike	1.05
Amount	2

We will plot the results for the ratio of expected return to value at risk, also known as RoRaC.

$$\text{RoRaC} = \frac{\mu - r}{\text{VaR}} \text{ or } \frac{\mu - r}{\text{CVaR}}$$

VaR, the value at risk, is mathematically defined as a quantile of the probability distribution of returns. In the plot below, we will use a quantile of 1%. In that case, our VaR is a minimum return that is missed with a probability of only 1%. One of the shortcomings is its ignorance towards anything that happens with a probability of less than 1%. The CVaR therefore averages all scenarios that fall below the 1% threshold, but still faces the problems described below.

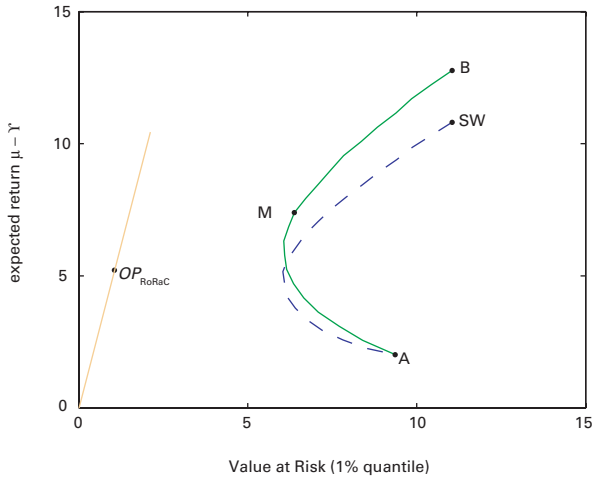
The second figure shows the RoRaC performance measure applied to the buy&hold portfolios, as well as to the already introduced stop-win strategy from the previous section. With the option portfolio OP_{RoRaC} we can limit the downside risk measured by VaR or CVaR and thus increase the RoRaC measure. In case of using CVaR, the risk measure of the option portfolio is the same, since all shortfall scenarios are capped at the same level. Only the plain stocks would exhibit slightly lower RoRaCs. However, it has been pointed out that CVaR solves some other shortcomings of VaR, which are beyond the scope of this document (RU00). Nonetheless, the possibility to manipulate the RoRaC measure is even bigger than in the Sharpe case.

Downside risk measures, such as VaR and CVaR, are often required by legal regulations to comply with official limits. This risk measure is easily optimised with stop loss trading and put options. However, these risk averse strategies provide only little profits and should not be exercised without reason. Industry wide it would lead to global pro-cyclic behavior, hence increasing volatility.

Economic theory

Economic equilibrium arguments provide some additional insight into the topic of optimal portfolios in efficient markets. If there existed a universal agreement over an optimal portfolio composition, and everybody agreed on the statistical parameters, then everybody would consequently try to hold a portfolio in exactly the same proportion as stocks are available on the market (ZB02). Due to the limited supply of stocks there is no other composition that could be held by every investor. Prices would adjust until the market portfolio is optimal under the agreed performance measure. Hence, only the

Fig 2. RoRaC performance measure



market portfolio can be globally optimal. Any formula or performance measure that suggests any other portfolio to be optimal, is either inappropriate or applies only to a specific group of investors with specific preferences.

Concerning the preferences of a company, it has been pointed out by Modigliani & Miller in their famous thesis, that exposure to efficient markets held by the firm is even irrelevant for its economic value

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(MM58). They argue that the shareholders of the firm are able to change the firm's exposure to market risk with additional transactions. For instance, if the company decides to sell a number of stocks, each shareholder could buy the proportional number to end up with the original composition.

This effect leads the way to a rough definition of a firm's preferences regarding efficient markets. As firms cannot create value by investing on the capital market, they have to be rather careful to destroy none. Hence, the main objective should be to reduce the so-called frictional costs, which could arise from financial distress or taxation issues. Firms have to be able to meet their liabilities with high confidence, which gives some indication of their optimal portfolio.

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Conclusion

Concerning the private investor we must conclude that no portfolio composition arguably exceeds a naively diversified mixture of available stocks. Simple mathematical formulas lack a reasonable motivation. Economists found utility functions as an expression for individual preferences. Although much more complex to set up, they are even used to formalise a much wider range of human decisions (GP98). We further recognised that only the market portfolio with asset proportions that are proportional to overall available quantity, could be argued as the globally optimal portfolio. However, the market portfolio is simply the portfolio held by an average investor and, therefore, gives no advice to an individual investor.

We emphasise the fact that the main goal should be to determine an objective function which fits to the business model and not to solve a numerical optimisation problem with respect to an arbitrarily chosen performance measure. Hence, to understand how a business creates value is the crucial point. First of all, firms create value by producing, or otherwise obtaining goods, for less than they can sell them. For instance, purchasing capital market products, rearranging them and selling the new mix charging a fee is a business model which adds value. Mutual funds are examples of financial intermediaries that provide virtually full divisibility in subdividing the individual unit size of the traded securities they hold. Customers of mutual funds are willing to pay a certain fee for this service. Also pension funds or life insurers, as well as investment banks, are engaged in capital markets to engineer their products. Indeed, most companies invest via capital markets to transfer economic resources in time, or to hedge their obligations linked to the capital market. In contrast to these examples, it is not an obvious business model to exchange risk for return via the capital market, as this service is already provided by the capital market itself in the most efficient way. The market is competitive enough that only differentially superior information and insight will earn money; the easy pickings have been picked. Similar considerations should enter into a discussion about an appropriate objective function.

In summary, we can conclude that managerial decisions cannot be easily rationalised and evaluated by inventing one simple performance measure. Applying a standard measure like RoRaC thoughtlessly to control a business could create seriously misleading incentives. L&P