Sergey Kolos and *Konstantin Mardanov* of Citi Commodities present a general framework for pricing contracts with volumetric risk. They concentrate on a simple payoff and focus the discussion on the main challenge of such contracts, the pricing of the unhedgeable risk due to uncertain volume

Pricing volumetric risk

★ One of the many challenges posed by commodity markets is the market incompleteness due to an exposure to unhedgeable volumetric risk in that volumes produced or consumed differ from the volumes planned.

Volumetric (quantity) risk has long been recognised in the agricultural literature. McKinnon (1967) considered a farmer (a primary producer in general) who wishes to stabilise his future income by protecting himself "from the vagaries of his own output as well as vagaries in market prices" and calculated the variance-optimising hedge ratio of futures.

Later Moschini & Lapan (1995) recognised that the nonlinearity of payoff in price calls for nonlinear hedging instruments and added options to the hedging portfolio available to a primary producer. Assuming constant absolute risk aversion (CARA) utility function and joint normality for the random variables involved, they found the exact hedging positions in futures and straddles, and studied the implications of having options in the hedging portfolio.

Brown & Toft (2002) expanded the set of hedging instruments even further by deriving an optimal payoff function for a value-maximising firm that faces both hedgeable price and unhedgeable quantity risk. The authors implicitly assumed risk-neutrality of the firm and reduced the incomplete market problem by integrating over the quantity risk.

Recently, Oum & Oren (2007) extended the approach by accounting for agent's risk preferences via a utility function and derived a closed-form optimal payoff function for a load serving entity obligated to supply electricity upon demand at a predetermined fixed price.

Kolos & Ronn (2004) considered finding optimal positions in vanilla option and a swap to hedge the downside risk while capturing the upside.

In this paper we consider a problem of pricing a load-serving full requirement swap. These contracts are popular with electric distribution companies that have responsibility to provide power to customers who pay a fixed price per unit of load. Electric distribution companies hedge these uncertain cashflows by entering contracts called full requirement swaps, which they procure through auctions (New Jersey Board of Public Utilities, 2004). Counterparties that acquire these swaps receive the following payoff:

$$\sum_i Q_i \left(K - P_i\right) \tag{1}$$

where Q_i is a quantity (load) served during some reference time period t_i, P_i is a market price for the period t_i and K is a fixed price (rate) set at the inception of the contract. The acquirer of the contract needs to estimate the fixed payment K that one is willing to receive for taking on the risk present in payoff (1). Two sources of risk present in the contract can be readily identified: one is due to the price and the other is due to the load. We assume that there is a liquid market of instruments written on the price so that the price risk can be hedged away completely. The quantity risk, however, cannot be completely hedged as there are no tradable instruments written on load.

In the next section we discuss an intuitive, *ad hoc* approach to hedging the payoff with futures and options where the strikes are set voluntarily. We then formulate the hedging problem that we solve. The results are then illustrated with two simple examples.

Ad hoc approach

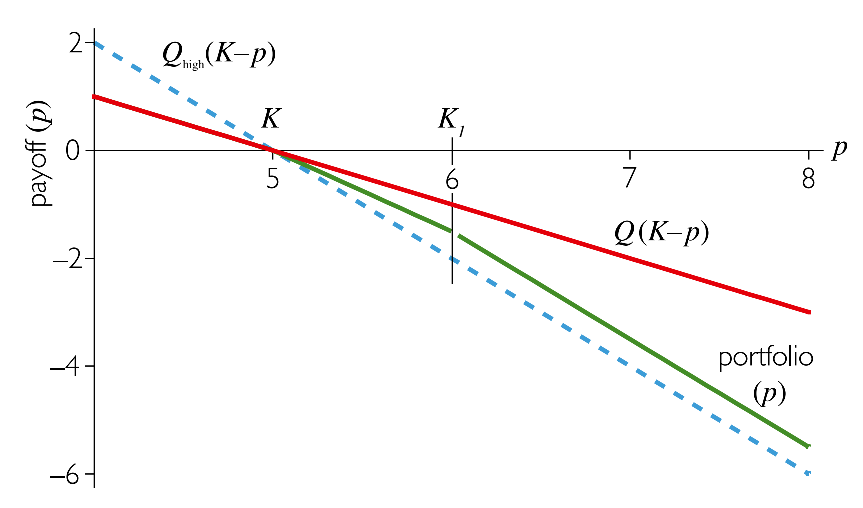
For simplicity, we limit the discussion to a one-period version of (1) and therefore assume that we are receiving the payoff:

$$Q\left(K - P\right) \tag{2}$$

where Q is a random quantity, P is a random variable price and K is a fixed price set at the inception of the contract. We also assume an interest rate of zero to facilitate the presentation. The pricing problem is to find a fair value for the fixed price K, which would make the price of the payoff zero at inception.

We start the construction of the *ad hoc* hedge portfolio by buying s swaps with the same fixed price K: $s(P-K)$. Then the combined payoff becomes: $(Q-s)(K-P)$. If we set s equal to the expected load, we get protection that will work on average, but not in all scenarios. The worst case is when both the load and the price go in the same direction, which can happen when correlation between price and quantity is not identically equal to –1. In power markets this correlation is usually positive as less efficient generators are used to supply higher loads.

To limit our exposure to the risk of price and quantity spikes, that is, the risk of high values of P and Q (the case of low load price can be analysed in a similar manner), we can buy h_0 call options with strike K to make the total payoff of the portfolio for high prices:



F1. *Ad hoc* portfolio payoff versus full requirement payoffs
The *ad hoc* portfolio payoff (green line) compared to full requirement payoffs when average load is realised (red line) and when high load Q_{high} (dashed line) is realised

$$(Q-s-h_0)(K-P)-h_0C_0$$

where C_0 is a value of a call option struck at K, and for $P>K$ the payoff of call option is $P-K$. The first term of this payoff is negative only when $Q>s+h_0$. Therefore, by buying a sufficiently large number of call options we can cover most of the load variability and thus ensure that we will not have any surprise losses. This approach works since a call option can be viewed as an option with binary volumetric payoff conditioned on price, so that when the price is above K, the call option adds a unit of quantity to the portfolio, thus hedging an adverse movement of quantity. Note that when quantity decreases while $P>K$, the call option still adds to now positive cashflow. This is analogous to a super-replication portfolio, which protects from downside risk, but can enhance upside. Such protection, however, may come at a significant cost as we may have to buy too many close to at-the-money options.

To reduce the cost, we can buy fewer options struck at K and also h_1 of cheaper, more out-of-the-money call options struck at $K_1>K$. They strengthen our protection when the load is between $s+h_0$ and $s+h_0+h_1$ by completely eliminating surprise losses at a cost of $h_1(K_1-K)+h_1C_1$; we exchange potentially unlimited surprise losses for losses of known size. When the price $P>K_1$, the total payoff of the portfolio is:

$$(Q-s-h_0-h_1)(K-P)-h_1(K_1-K)-h_0C_0-h_1C_1$$

This payoff (ignoring initial costs C_0 & C_1) is shown in Figure 1.

The above discussion shows the rationale behind a common market practice to hedge a load-following contract by buying futures to cover the expected load, and straddles and strangles to limit the losses incurred when load and price move in the same direction. The fixed rate K is chosen so that the expected value of the portfolio is zero. The risk preferences implicitly affect the choice of strikes and volumes for strangles. In the next section we formalise how strikes and volumes are selected given risk preferences.

Pricing and hedging portfolio

Recall that we want to hedge the payoff:

$$Q(K-P)$$

where Q is a random quantity, P is a random variable price and K is a fixed price (rate) set at inception of the contract. The pricing problem is to find a fair value for the fixed price K.

Let us consider a hedging portfolio consisting of h_1 options with strikes K_i and a cash position b. We note that having only calls and cash in a portfolio is not a restriction since one can always replicate puts and swaps by combinations of calls and cash. Then the combined value of the payoff and the hedging portfolio is the sum of the components: the contract payoff, the hedge payoff and the cost of the hedges, as shown here:

$$Q(K-P)+\left(b+\sum_i h_i(P-K_i)_+\right)-\left(b+\sum_i h_iC_i\right)$$

where C_i is the value of a call option struck at K_i. Given that the terminal underlying price is equal p, the terminal value is:

$$(Q|P=p)(K-p)+\sum_i h_i\left((p-K_i)_+-C_i\right)$$

Let conditional distribution be $(Q|P=p)=q(p)+\varepsilon_p$, where $q(p)=E[Q|P=p]$ is a deterministic function and ε_p is a random variable with zero expectation. In the limit $\Delta K=K_i-K_{i-1}\to 0$, we have $h(K_i)\Delta K=h(s_i)ds$ and the payoff for a given price $P=p$ becomes:

$$\int_0^\infty h(s)\left((p-s)_+-C(s)\right)ds+q(p)(K-p)+\varepsilon_p(K-p)$$

Suppose we want to set up a portfolio that would hedge the downside risk due to the random term $\varepsilon_p(K-p)$ to an α-th percentile probability of loss. Depending on whether p is greater than K or not, we have two cases: if $p>K$, then the worst outcomes happen when ε is large so that the quantity of interest is the $(1-\alpha)$-th percentile of ε_p, which we denote as $\xi_{p,1-\alpha}$. Otherwise, if $p<K$, the worst cases correspond to small instances of ε, which brings us to $\xi_{p,\alpha}$, the α-th percentile of ε_p.

Therefore, for $p>K$ the hedging portfolio should satisfy:

$$\begin{aligned}&\int_0^\infty h(s)\left((p-s)_+ - C(s)\right)ds \\ &+q(p)(K-p)+\xi_{p,1-\alpha}(K-p)=0\end{aligned} \quad (3)$$

and for $p \le K$:

$$\begin{aligned}&\int_0^\infty h(s)\left((p-s)_+ - C(s)\right)ds \\ &+q(p)(K-p)+\xi_{p,\alpha}(K-p)=0\end{aligned} \quad (4)$$

Defining $\xi(p,K)$ as:

$$\xi_\alpha(p,K)=\begin{cases}\xi_{p,\alpha}, \text{ if } p<K \\ \xi_{p,1-\alpha}, \text{ if } p \ge K\end{cases} \quad (5)$$

we can combine (3) and (4) into one equation:

$$\begin{aligned}&\int_0^\infty h(s)\left((p-s)_+ - C(s)\right)ds \\ &=-\left(q(p)+\xi_\alpha(p,K)\right)(K-p)\end{aligned} \quad (6)$$

To find the rate K, we take the mathematical expectation of both sides of (6) over $P=p$ under the risk-neutral measure. Noticing that:

$$E\left((p-s)_+\right)=C(s)$$

we get equation on K given risk preference α:

$$KE\left[q(p)\right]-E\left[pq(p)\right]+E\left[\xi_\alpha(p,K)(K-p)\right]=0 \quad (7)$$

To find a hedging portfolio, we first consider the boundary $p=0$ to obtain

$$\int_0^\infty h(s)C(s)ds = K\left(q(0)+\xi_\alpha(0,K)\right)$$

That is, if the terminal price is zero, all calls that we bought have zero payoffs yet we still bear their cost. Note that the right-hand side is identically zero if we will not be supplying load at zero price, i.e. if $q(0)=0$. This is what we assume for now, but it is straightforward to accommodate a case of a non-zero load and a zero price (which is quite possible in power markets wherein price can even become negative with still positive output).

Therefore we have:

$$\int_0^\infty h(s)(p-s)_+ ds = -\left(q(p)+\xi_\alpha(p,K)\right)(K-p) \quad (8)$$

Now we notice that the integrand is such that we can set the upper limit of integration to p and twice differentiate both sides of the equation with respect to p to get

$$h(s)=-F''(s) \quad (9)$$

where

$$F(p)=\left(q(p)+\xi_\alpha(p,K)\right)(K-p) \quad (10)$$

Although one could, in principle, use (9) to obtain the hedge $h(s)$, it is more convenient to observe from (8) that the hedge is set up to offset the adjusted payoff (10). Recall that K represents the acceptable fixed price for the contract, which is computed by solving equation (7). We can use a representation of a general continuous payoff in terms of European puts and calls (see *Appendix A*) to determine the replicating portfolio $w(k)$. Then the hedging portfolio is the counter side of the replicating portfolio:

$$h(k)=-w(k)$$

To summarise, let us retrace the main steps of the pricing procedure. First, we assume that from historical data or other means we know the conditional expectation of the load given the price, $q(p)=E[Q|P=p]$ and the distribution of the load noise, ε_p. Then we choose α, the accepted probability of a loss on the hedged portfolio. Lastly, we calculate the fixed rate K by solving (7) and obtain the hedging strategy by replicating the adjusted payoff (10). Then we can calculate the statistics of the hedged profit-and-loss (P&L) distribution such as expected P&L, expected loss and expected profit (see *Appendix B*). In practice, one may want to run a few scenarios with different α's and choose the one that produces an acceptable P&L distribution.

Example 1

Assume that the terminal price is lognormally distributed with mean p_0 and standard deviation σ. Then the value of a payoff $g(p)$ is given by the expectation:

$$E\left[g(p_T)\right]=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} g\left(p_0 e^{-\frac{1}{2}\sigma^2 T+\sigma\sqrt{T}x}\right)e^{-\frac{1}{2}x^2}dx \quad (11)$$

For this example, we set the price, $p_0=5$, the price volatility, $\sigma=40\%$ and the maturity, $T=1$.

Assume a simple functional form for the load as a function of price – for example, a logarithmic function $q(p)=\ln(4p+1)$ and the noise term with the standard deviation proportional to price:

$$Q(p)=\ln(4p+1)+p\varepsilon$$

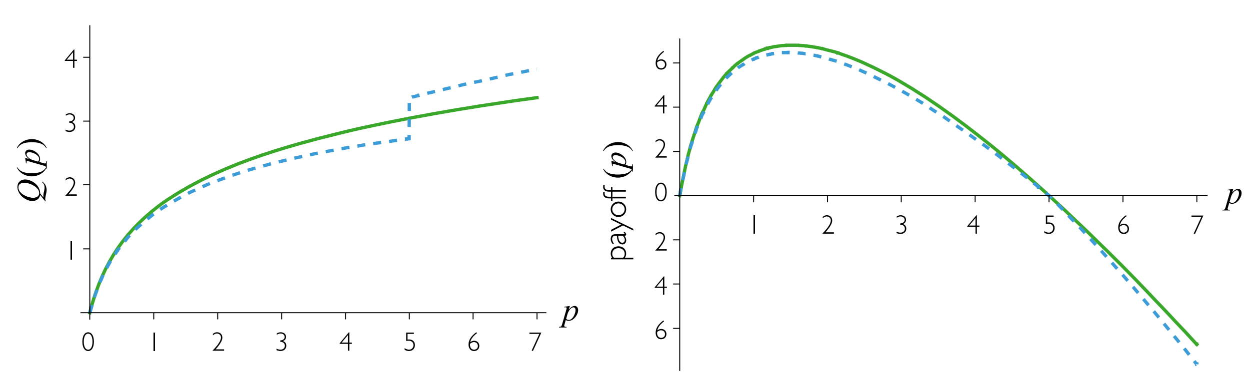
where the noise term ε is normally distributed with zero mean and standrad deviation σ_ε:

$$\varepsilon \sim N(0,\sigma_\varepsilon)$$

Then the α^{th} percentile of the load function is given by:

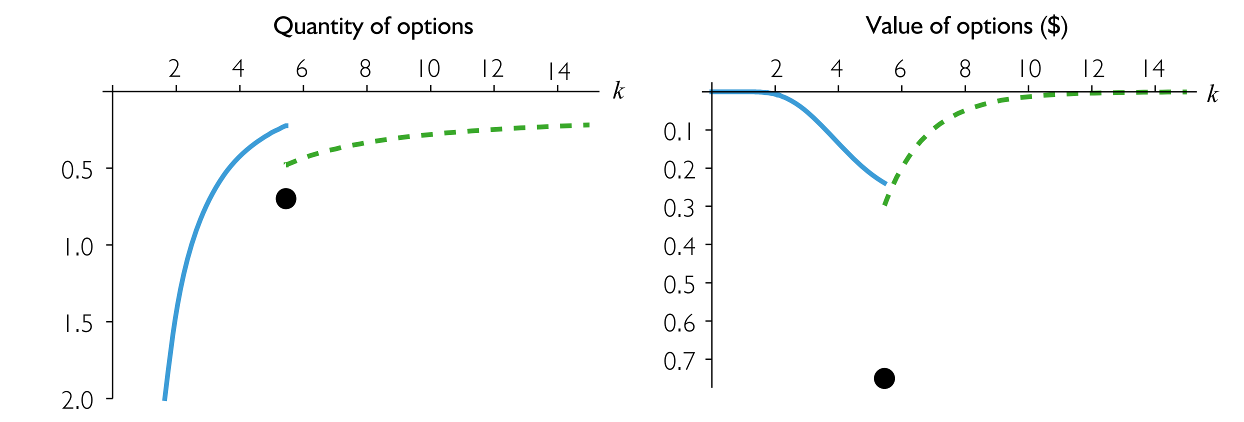
$$\xi_\alpha = p\sigma_\varepsilon\sqrt{2}erf^{-1}(2\alpha-1)$$

The function $\xi_\alpha(p,K)$ is then constructed using definition (5).



F2. Assumed dependence of the load on price

On the left, the load $q(p)=\ln(4p+1)$ is shown with solid line and its 10-th percentile, $q(p)+\xi_{10\%}(p,K)$, with dashed line. The standard deviation of the noise, σ_ε, was set to 5%. On the right, the expected payoff $q(p)(K-p)$ is shown with solid line, and the adjusted payoff $(q(p)+\xi_{10\%}(p,K))(K-p)$, with dashed line. The rate K was tentatively set to 5 for illustration only



F3. Adjusted portfolio details

Details of the adjusted portfolio for the adjusted payoff with the fixed rate $K_{10\%}=5.452$. The graph on the left shows the quantities of options $(w(k))$ struck at k (solid line for puts and dashed line for calls). The graph on the right shows the values of the positions in puts and calls. The total values of the option holdings are -1.148 and -0.423, respectively, which is exactly offset by the value in forwards with the delivery price $K_{10\%}$

We show the assumed dependence of the load on price, $q(p)$ along with its 10-th percentile on the left part of figure 2. The expected payoff $q(p)(K-p)$ and the adjusted payoff $(q(p)+\xi_{10\%}(p,K))(K-p)$ are shown on the right part of Figure 2 for the rate K tentatively set to 5.

Strike and hedging portfolio

We account for the residual risk by selecting α using our preferences, and solving equation (7). If we set $\alpha=10\%$ we find:

$$K_{10\%}=5.452$$

We find a replicating portfolio for the adjusted payoff (10) by the procedure described in *Appendix A*. The result is shown in figure 3.

Table 1 reports the residual risk in the adjusted portfolio by the expected PnL, the expected loss and the expected gain (see *Appendix B* for formulas).

Dependence on risk preferences

If we are risk neutral, we can neglect the randomness in load and determine the rate K_{exp} by setting the value of the expected payoff $g(P)=q(p)(K-p)$ to zero:

$$E\left[q(p)(K-p)\right]=0$$

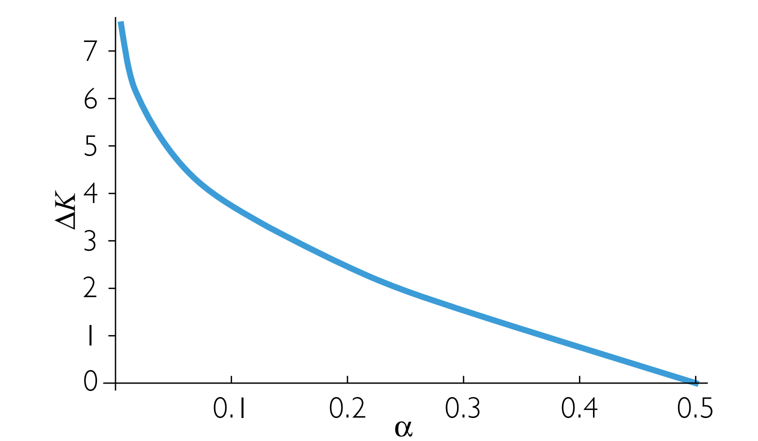
thereby obtaining

$$K_{exp}=\frac{E\left[q(p)p\right]}{E\left[q(p)\right]} \quad (12)$$

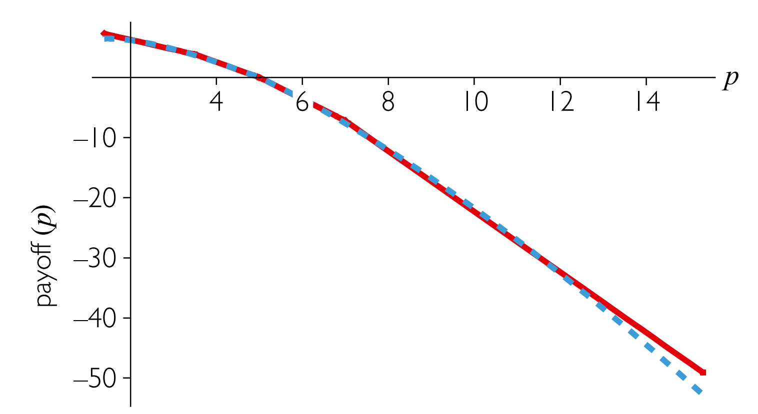
T1. The residual volumetric risk of the adjusted portfolio

If the probability of a loss is set to 10%, then the portfolio is expected to lose on average 1.829 (with the probability of 10%) and gain on average 0.853 (with the probability of 90%). On average, the portfolio makes:
0.9 x 0.853 – 0.1 x 1.829 = 0.584

Statistic	Value
Expected profit-and-loss (P&L)	0.584
Expected loss	–1.829
Expected profit	0.853



F4. Fixed rate and loss probability
The percentage change in the fixed rate, $\Delta K \equiv (K_\alpha - K_{exp})/K_{exp}$, as a function of the loss probability



F5. Estimating payoff and adjusted payoff
Approximating payoff (P) (solid line) and adjusted payoff $g(P)$ (dashed line)

Evaluating the expectations by means of (11), we obtain:

$$K_{exp} = 5.255$$

Figure 4 shows how strike K_α changes from risk neutral value K_{exp} as we change risk preferences expressed by the probability of loss α. We notice how the rate increased by about 4% from 5.255 to 5.452 reflecting our desire to be compensated for the risk due to load uncertainty.

Finite number of strikes

In practice only a few strikes are available for trading so the assumption of a continuum of strikes implied by the theory may seem too accommodating. In this section we show how one can form an approximate hedge out of just three strikes.

Consider a portfolio of s swaps, w_0 put options with strike K, w_1 put options with strike K_1 and w_2 call options with strike K_2. The payoff of this portfolio is:

$$payout(P) = s(P-K) + w_0(K-P)_+ + w_1(K_1-P)_+ + w_2(P-K_2)_+$$

In order to find the weights s, w_0, w_1 and w_2 that approximate the adjusted payoff $g(P)$ as close as possible, we minimise a weighted squared difference between the payoffs, with weights set equal to the probability density of price:

$$\int_{-\infty}^{\infty} \frac{e^{-\frac{\varepsilon^2}{2}}}{\sqrt{2\pi}} \left(g\left(p_0 e^{-\frac{1}{2}\sigma^2 T + \sigma\sqrt{T}\varepsilon}\right) - payout\left(p_0 e^{-\frac{1}{2}\sigma^2 T + \sigma\sqrt{T}\varepsilon}\right)\right)^2 d\varepsilon$$

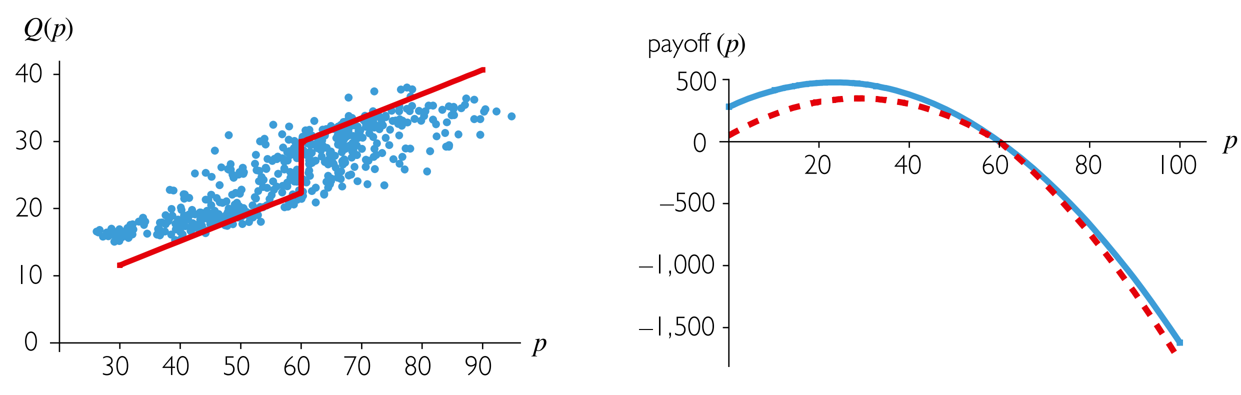
For the strikes $K = K_\alpha$, $K_1 = 3.5$, $K_2 = 7$, the resulting weights are: $s = -3.539$, $w_0 = -0.966$, $w_1 = -1.111$ and $w_2 = -1.415$.

Figure 5 compares the original payoff and its approximation. One can see that the approximation deteriorates as price goes beyond about two standard deviations. However, the probability of such events quickly vanishes so that overall the approximate hedge is expected to perform close to the theoretical one. We also note that the problem of the optimal choice of weights and strikes for a discrete payoff approximation deserves a separate research and will not be discussed in this paper.

Example 2

Now we illustrate the approach using a simple example drawn from a load profile of a US electric distribution company located in ISO New England. We assume that on November 1, 2007 we need to price a full requirement contract for July 2008. The scatter plot on the left side of figure 6 shows an average daily load for the month of July. For simplicity, we assume a linear relationship between load and price, $q(p) = a_0 + a_1 p$ and find the coefficients by regressing load Q_i on price p_i to obtain:

$$q(p) = 4.66 + 0.36p$$



F6. Historical daily load, price, risk expected and adjusted payoff

The scatter plot on the left represents historical average daily load Q versus price (p), along with the 10-th percentile load function, $q(p)+\xi_{10\%}(p,K)$. On the right, the solid line represents the risk expected payoff $q(p)(K-p)$, and the dashed line, the adjusted payoff $(q(p)+\xi_{10\%}(p,K))(K-p)$. The rate K was tentatively set to 60 for illustration only *Source: Citi Commodities*

We assume risk preferences are given by the 10% probability of a loss. Assuming that distribution of residual errors $e_i \equiv Q_i - q(p_i)$ is independent of price level p_i we find 10% and 90% quantiles of residual errors:

$$\xi_{10\%} = -3.84, \quad \xi_{90\%} = 3.72$$

and form the function $\xi_\alpha(p, K)$ using definition (5). The resulting 10-th percentile load function and adjusted payoff are shown on Figure 6.

Assuming price is distributed lognormally with current market values for July 2008 forward price $P_0 = 90$ \$/MWh and volatility $\sigma = 30\%$, we can compute expectation of a payoff function using (11). Using (12), the risk neutral strike is:

$$K_{\exp} = 96.1\$ / MWh$$

Solving equation (7) we obtain risk adjusted strike:

$$K_{10\%} = 98.3\$ / MWh$$

As in 'Example 1', we observe the rate increase by about 2%, which is our compensation for bearing the volumetric risk.

Conclusion

Volumetric risk is a peculiarity of commodity markets that leads to a market incompleteness and therefore requires a special consideration. In this paper we have concentrated on a simplified version of a load-following contract whose payoff is of the form:

$$Q(K - P)$$

where Q is a random quantity, P is a random variable price and K is a fixed price to be set at the inception of the contract. The pricing problem is to determine the fair value for the fixed price K.

We reduced the question of the trader's risk preferences to the choice of the probability of a loss that a trader is willing to accept. Having accounted for the risk preferences, we determined the payoff of a hedging portfolio that limits the downfall risk to the specified level. Since the payoff of the portfolio depends only on price, it can be priced via a standard no-arbitrage technique. Throughout our discussion we have kept the downfall probability as simple as just one good-for-all number. However, it is straightforward to allow the probability depend, for example, on the price of the underlying thereby incorporating more information on trader's risk preferences. In practice, one may want to run a few scenarios with different values of downfall probability and then choose the one that produces an acceptable profit-and-loss distribution.

We have illustrated our approach with two simple examples. In the first example we postulated the distributions of the load and the price of the underlying, and priced the payoff with and without a consideration for the residual unhedgeable risk. In the second example we estimated the distribution of the load and price using the historical relationship between price and load and adjusting it by using forward looking marking information on price distribution given by futures and option prices. The results quantified the intuitively obvious premise that one should be compensated for bearing a risk that cannot be hedged.

The approach can be readily generalised to natural extensions of (2) such as a multi-period payoff, a payoff with a prepayment or a partial cash payment, a load with a strong seasonal component, and so forth. ER

Acknowledgements

The authors acknowledge helpful comments and suggestions of members of Citi Commodities' Quantitative Research.

Sergey Kolos, vice-president, Citi Commodities and Konstantin Mardanov, associate, Citi Commodities & PhD candidate at ICES, The University of Texas at Austin. Emails: Sergey.P.Kolos@citi.com, Konstantin.Mardanov@citi.com

Appendix A: Payoff decomposition

General European payoff

The replication of a general European payoff by put and call options with different strikes was derived by Carr & Madan (2001). For the purposes of this paper, we restrict our attention to payoffs represented by continuous piecewise twice differentiable functions and obtain the following representation:

$$f(y) = f(K) + f'(K+0)(y-K)_+ - f'(K-0)(K-y)_+ + \sum_{i=0}^{i_K-1}\left([f']_i (k_i - y)_+ + \int_{k_i}^{k_{i+1}} f''(k)(k-y)_+ \, dk\right) + \sum_{i=i_K}^{N-1}\left([f']_{i+1}(y-k_{i+1})_+ + \int_{k_i}^{k_{i+1}} f''(k)(y-k)_+ \, dk\right)$$

where $[f']_i \equiv f'(k_i+0) - f'(k_i+0)$, the jump of f' at k_i, and $k_0 = 0$, $k_{i_K} = K$ (an arbitrary positive number) and $k_N = \infty$.

Using approach described in Carr & Madan (2001) and integrating by parts the resulting expressions to get rid of the second derivative of f, we obtain:

$$V[f] = e^{-rT} f(K) - \sum_{i=0}^{i_K-1} \int_{k_i}^{k_{i+1}} f'(k)P'(k)dk - \sum_{i=i_K}^{i_N-1} \int_{k_i}^{k_{i+1}} f'(k)C'(k)dk$$

If the payoff f is a piecewise linear function, we define $f'(k)|_{k\varepsilon(k_i,k_{i+1})} = f_i'$, $P_i = P(k_i)$ and $C_i = C(k_i)$, and obtain:

$$V[f] = e^{-rT} f_{i_K} - \sum_{i=0}^{i_K-1} f_i'(P_{i+1} - P_i) - \sum_{i=i_K}^{N-1} f_i'(C_{i+1} - C_i)$$

Appendix B: Distribution of residual risk

Expected P&L

Profit-and-loss (P&L) of the contract together with a hedge is given by (8). Taking expectations over ε_p and p we find that:

$$E[\text{P\&L}] = K_\alpha E[q] - E[pq] = E\left[\xi_\alpha(p,K)(K-p)\right]$$

Expected loss

The residual PnL is negative when:

$$\varepsilon_x < \xi_{x,\alpha}, \text{ if } x < K$$
$$\varepsilon_x > \xi_{x,1-\alpha}, \text{ if } x \geq K$$

Then the expected loss is given by:

$$EL \equiv E\left[\text{P\&L} \mid \text{P\&L} < 0\right] = E\left[\bar{\xi}(p,K)(K-p)\right] - E\left[\xi_\alpha(p,K)(K-p)\right]$$

where

$$\bar{\xi}(x,K) = \begin{cases} \underline{\xi}_{x,\alpha} \equiv E\left[\varepsilon_x \mid \varepsilon_x < \xi_{x,\alpha}\right], \text{ if } x < K \\ \bar{\xi}_{x,1-\alpha} \equiv E\left[\varepsilon_x \mid \varepsilon_x > \xi_{x,1-\alpha}\right], \text{ if } x \geq K \end{cases}$$

Expected gain

Using the relationship:

$$E[\text{P\&L}] = P[\text{P\&L} > 0]E\left[\text{P\&L} \mid \text{P\&L} > 0\right] + P[\text{P\&L} < 0]E\left[\text{P\&L} \mid \text{P\&L} < 0\right]$$

the expected gain is:

$$EG \equiv E\left[\text{P\&L} \mid \text{P\&L} > 0\right] = \frac{E[\text{P\&L}] - \alpha EL}{1-\alpha}$$

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