Some experts are questioning whether mean-reversion models – once the standard way of explaining energy price movements after simple one-factor Brownian motion modelling fell from favour – are still suited to the petroleum and gas markets. *Dario Ghazi* and *Sam Sivothayan* from City Practitioners consider whether mean reversion still dominates

Has mean reversion had its day?

Dario Ghazi (above) and Sam Sivothayan from City Practitioners

★ A variety of energy pricing models have been developed since the 1970s. The first were replications of models already in use in the financial markets, based on Brownian motion and random walk, the theory that historic price moves can't be used to forecast prices.[1] Eduardo Schwartz, for example, selected the geometric Brownian motion of Black-Scholes-Merton (1973) to explain the spot price process in his 1990 paper. The early one-factor geometric Brownian motion energy commodities models suggested that prices would fluctuate randomly but would grow on average. In such conditions, it can be suggested that investors will get the best returns from a hold-and-buy strategy – something that did not seem to occur in the 1990s, as prices did not grow on average over this period.

Therefore during the 1990s, dissatisfaction grew with geometric Brownian motion as a means of modelling energy spot prices.

The quest for a more appropriate method of modelling energy prices led academics and others to focus on mean reversion: the theory that prices revert to a certain level following price volatility. In the case of energy prices, this level is the long-term cost of production.

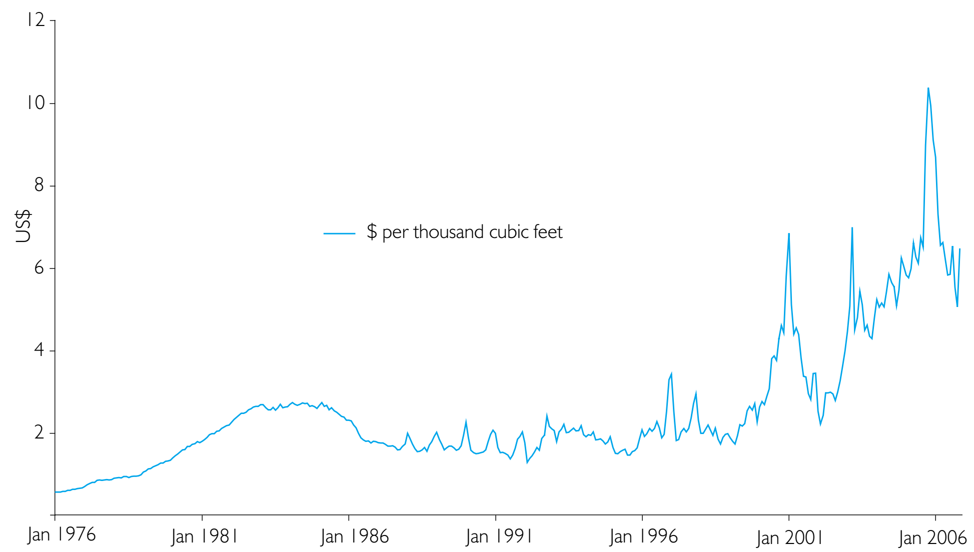
Mean reversion was popularised by Oldrich Alfons Vasicek who, in his 1977 paper, made use of the theory to explain interest rate movements. As a result, several mean-reversion models where introduced during the 1990s – for example, Heston (1993) and Schwartz (1997). More recently, however, opinion appears to have swung in the opposite direction, with doubts now being voiced as to whether energy commodities price trajectories do in fact mean revert. Indeed, some experts suggest that the mean-reversion characteristic may actually have faded out of the petroleum and gas markets in the years since 2000.

Mean reversion – dead or alive?

So what really is happening to the mean-reversion characteristic? Are we actually seeing it fade? First, mean reversion still remains strongly evident in markets such as electricity, where prices do not evolve freely to any level but generally gravitate towards the long-run price (that is, the long-run production cost level). However, in the gas and oil markets, the story is very different, as figures 1 and 2 indicate.

Figure 1 shows the US natural gas monthly price for the period January 1976 until November 2006 provided by the US Energy Information Agency (EIA). For the period January 1976 until mid-1999 there is clearly a mean-reverting price level around $2 per thousand subic feet. However, during mid-1999 the price level seems to break and at the beginning of 2002 the natural gas prices take off, attaining levels of just above $5 per

1 Brownian motion is defined as the movement paths of pollen particles suspended in water that the scientist Robert Brown observed in 1827. Random walk theory became popular in 1973 when Burton Malkiel published his book A Random Walk Down Wall Street *which essentially showed that the past stock price movements or market trends could not be used for forecasting purposes successfully.*



F1. US natural gas wellhead price for US natural gas

Natural gas time series: monthly data. Wellhead price ($ per thousand cubic feet) = GAS
Sample period: January 1976 to November 2007 (371 observations); LAS = ln (GAS);
LGAS_SA = seasonally adjusted LGAS

thousand cubic feet in October 2006.

Figure 2 shows the US crude oil daily spot price for the period January 2, 1976 to January 17, 2007, as provided by the EIA. For the period from early-1976 until early-2000 the oil price appears to mean revert at a level of around $20 a barrel, however, since then the oil price has launched at new higher levels and set at $60.77 a barrel at January 2, 2007. As figures 1 and 2 indicate, both the crude oil and the natural gas prices increase suddenly in early 2000 for a two-year period and then bounce back to the old level, only to launch into new and higher price trajectories.

The above phenomenon has been analysed by commentators, notably Hélyette Geman (2005, 2006) and Pindyck (2001). In her 2006 paper, Geman considers price movements for US oil and gas prices during the years 1994-2004. Using datasets from the New York Mercantile Exchange (Nymex), she runs two well established unit root tests, the Augmented Dickey-Fuller and the Phillips-Perron in order to find the periods for which mean reversion, in the classical sense, prevail in oil and gas spot prices, and also to ascertain whether there are periods during which the mean-reversion effect appears to have faded away. Her findings demonstrate that a mean-reversion pattern occurs for the oil spot price over the period 1994 to 2000. However, this then changes to a random walk process for the 2000 to 2004 period. In the case of natural gas prices, the mean-reversion effect prevails up until 1999 but then after 2000 it too changes to a random walk process.

To test the Geman hypothesis further, we ran the same unit root tests for Nymex oil spot prices and Nymex natural gas, taking a sample period of 1986 to 2007 for oil spot prices and 1976-2006 for natural gas.

The Augmented Dickey-Fuller (ADF) and the Phillips-Perron (PP) tests are the most

F2. Cushing, OK WTI spot price FOB

Oil time series: daily data. Cushing, OK WTI spot price FOB = OILSPOT
Sample period: January 2, 1976 to January 17, 2007 (5310 observations); LOIL = ln (OIL SPOT)

T1. Unit root tests' p-values[1] for Nymex oil & gas prices

Time Series	Sample Period	Augmented Dickey-Fuller	Phillips-Perron
Oil Spot Price	02/01/1986 – 6/02/2007	0.69	0.71
	02/01/1986 – 1/12/1998	0.38	0.34
	04/01/1999 – 6/02/2007	0.91	0.93
Natural Gas	01/1976 – 09/2006	0.31	0.41
	01/1976 – 12/1998	0.16	0.18
	01/1999 – 09/2006	0.76	0.76

[1] *MacKinnon (1996) one-sided probability values*

popular autoregressive unit root tests used for financial series. They consist of estimating an AR(p) model and performing a t-test in order to search for a unit root in the regression equation linking Xt and its lags. A high probability value supports the null hypothesis of a unit root; therefore, the process is stationary and follows a random walk pattern, in other words, Brownian motion. If the p-value is significantly below 1 and closer to 0, then the process is mean reverting.

The results of our tests, as outlined in table 1, appear to indicate that oil prices actually reject the mean-reverting structure for the whole period of 1986–2007, and especially over the more recent period of 1999–2007. In the case of gas, while the seasonality of prices blurs the signals, price levels clearly mean revert during the years of 1976 to 1998, followed by a significant breaking down of the characteristic during 1999 to 2006.

An evolving characteristic

While the above findings do indicate that mean reversion has faded (at least in its classical sense), we should not be too hasty in writing it off. What appears likely to have happened is that mean reversion has undergone a change in form. Re-evaluation of the characteristic and the way in which it is incorporated into pricing models is therefore required.

And recent energy pricing literature indicates that experts are doing just that. While the original pricing models for energy (Gibson and Schwartz, 1990) used the geometric Brownian motion process, these methods evolved to two- and three-factor mean-reverting models, for example, as used by Schwartz (1997, 2003) and Geman and Nguyen (2005), or to models mixing mean reversion with jumps as seen in Cartea and Figueroa (2005). In addition, Geman (2006) presented a three-state variable model, which incorporates stochastic volatility and links the spot price mean-reversion with a long-run level for oil driven by a geometric Brownian motion process:

$$dS_t = a(L_t - S_t)S_t dt + \sigma_t S_t dW_t^1,$$
$$dy_t = a(b - y_t)dt + \eta\sqrt{y_t}\,dW_t^2,$$
$$dL_t = \mu L_t dt + \xi L_t dW_t^3.$$

Where:

$$y_t = \sigma_t^2$$

The Brownian motions W^1, W^2 and W^3 are positively correlated.

The positive correlation between W^1 and W^2 can be translated as the 'inverse leverage' effect that is commonly observed in commodity markets (otherwise known as the forward curve backwardation phenomenon), as shown by Litzenberger and Rabinowitz (1995). The implication of the positive correlation between W^1 and W^3 is that the long-run mean-reverting level can fluctuate, as well as the short run. Practically speaking, this model gives a greater amount of flexibility for energy commodities price formation, allowing for both the short- and the long-term mean-reverting levels to change, while at the same time preserving the fundamental characteristics of price mean-reversion and stochastic volatility.

Another three-factor parsimonious mean-reverting model is that of Cortazar and Schwartz (2003) for oil spot and future price dynamics. This model allows for both the short- and the long-run price return levels to be stochastic:

$$dS_t = (\nu - y)Sdt + \sigma_1 SdW_1$$
$$dy = -\kappa y dt + \sigma_2 dW_2$$
$$d\nu = a(\bar{\nu} - \nu)dt + \sigma_3 dW_3$$

Here y is the demanded convenience yield and ν expresses the expected long-term spot price return.

The volatilities σ_2 and σ_3 are constant. The parameter a is also constant: it's the speed of mean reversion. The standard Brownian motions W_i are correlated as follows:

$$dW_1 dW_2 = \rho_{12} dt$$
$$dW_1 dW_3 = \rho_{13} dt$$
$$dW_2 dW_3 = \rho_{23} dt$$

In this model the third factor is the long-run spot price return, ν, which is stochastic and mean-reverts to a level $\bar{\nu}$ with the mean-reversion co-efficient (speed to mean- reverting value) α and volatility σ_3. The implication of this model is that oil prices fluctuate as a result of variation both to short-term factors such as inventory changes and long-term factors such as technological changes. The model provides greater flexibility in the shape of oil future curves. Cortazar and Schwartz (2003) claim that the model has been used for a number of years by an oil company in order to provide an estimate of the term structure of oil future prices. It is also used by the website *www.riskamerica.com* for daily estimates of the oil and copper futures curves.

Flexible models required

In the years following 2000, mean reversion has undoubtedly suffered significant disruption. Energy prices, driven by a variety of factors, such as geopolitics and fears over dwindling supplies of fossil fuels, have risen sharply and, at least for the present, appear likely to remain high. Undoubtedly the cost of production has also risen, but not to the extent where it can entirely explain the run-up in prices since 2000.

Therefore it would appear that prices have moved away from being tied to the cost of production which, in turn, has led to speculation that the mean-reversion characteristic may have faded out of certain energy markets or, at the least, that mean reversion modelling should be more dynamic allowing for greater model flexibility.

Rather than having disappeared altogether, mean-reversion would seem to have changed in form, and so new models are required to approximate mean-reversion in its evolved state. In response, more flexible models are being developed that allow for fluctuation in both short-term and long-term mean-reverting levels. ER

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