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Research Paper

Adapting the Basel II advanced internal-ratings-based models for International Financial Reporting Standard 9

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ABSTRACT

Banks around the globe are implementing International Financial Reporting Standard 9 (IFRS 9), which is a considerable effort. A key element of IFRS 9 is a forward-looking "expected loss" impairment model, which is a significant shift from the incurred-loss model. We examine how we may use advanced internal-ratings-based (A-IRB) models in the estimation of expected credit losses for IFRS 9 purposes. We highlight the necessary model adaptations required to satisfy the new accounting standard. By leveraging on the A-IRB models, banks can lessen their modeling efforts in fulfilling IFRS 9 and capture the synergy between different modeling endeavors within institutions. In outlining the proposed probability of default, loss given default and exposure at default models, we provide detailed examples of how they may be implemented on secured lending. Moreover, in discussing the issues related to the estimation of the expected credit loss for IFRS 9, we highlight the challenges involved and propose practical solutions to deal with them. For instance, we propose the use of

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a convexity adjustment approach to circumvent the need for assigning probabilities in multiple-scenario analysis.

Keywords: advanced internal-ratings-based (A-IRB) approach; IFRS 9; probability of default (PD); loss given default (LGD); exposure at default (EaD); expected loss (EL).

1 INTRODUCTION

The recent global financial crisis highlighted the deficiency of the existing accounting standard, International Accounting Standard 39 (IAS 39), in the use of an incurred-loss model, which is deemed to be backward-looking, to account for credit losses on loans and other financial instruments (Ernst & Young 2014). In the pursuit of a more timely recognition of credit loss in financial statements, the International Accounting Standards Board (IASB) has introduced a forward-looking expected credit loss model in a new accounting standard, "IFRS 9 Financial Instruments" (IFRS 9), to be adopted not later than January 1, 2018.[1] Unlike in IAS 39, where credit losses are only recognized upon the occurrences of credit events, IFRS 9 requires lenders (or asset holders) to recognize expected credit loss over the life of financial instruments. Moreover, the expected credit losses are to be measured on either a (forward-looking) twelve-month or a lifetime basis, depending on whether there has been a material increase in credit risk since the initial recognition. With its forward-looking nature and its new "three-bucket" approach (see Section 2 for more details), not only will credit losses be recognized earlier but more losses will potentially be recognized. The implementation of IFRS 9 is expected to result in an increase in the overall credit loss allowances of many banks and to have important implications for the regulatory capital requirements of such financial institutions (Deloitte 2013; Ernst & Young 2014).

Due to the tight implementation deadline and potentially significant implications, banks around the globe are deploying many of their resources in developing the necessary forward-looking expected loss (EL) impairment model, which is a significant shift from the current incurred-loss model. There is a certain degree of subjectivity, and considerable judgement needs to be made in developing and implementing these models. Different modeling approaches have been proposed by the accounting community (see, for example, Global Public Policy Committee 2016).[2] An industry

[1] IASB published the final version of IFRS 9 in July 2014. It replaced earlier versions of IFRS 9 on classification and measurement requirements (introduced in 2009 and 2010) and a hedge accounting model (introduced in 2013).

[2] Given the important implications of the new standard for the capital requirement of financial institutions, the Bank for International Settlements also provides supervisory guidance on the accounting for expected credit losses (Basel Committee on Banking Supervision 2015).

practice is yet to be established for the estimation of expected credit losses in satisfying IFRS 9.[3] Given the similarities between the IFRS 9's credit risk measure and that required to satisfy the Basel Committee on Banking Supervision's (BCBS's) regulatory requirement, a pragmatic solution is for banks to build on their internal models under the advanced internal-ratings-based (A-IRB) approach and leverage their well-established credit risk stress testing models to satisfy the IFRS 9 modeling needs. In this paper, we examine how we can utilize a suite of A-IRB models to estimate both one-year and lifetime expected credit losses for IFRS 9. Specifically, we adapt the A-IRB probability of default (PD), loss given default (LGD) and exposure at default (EaD) models for IFRS 9 use and show how we can arrive at the EL measure by integrating the PD, LGD and EaD parameters obtained from these models. To ensure the EL measure can reflect the current state of the economy and business cycle, the particular kind of model we examined is specifically developed to be dynamically driven by key macroeconomic variables. This kind of time series conditional model is commonly used to fulfill the stress testing requirement under Basel II (see, for example, Blümke 2010; Miu and Ozdemir 2009; Ozdemir and Miu 2008; Simons and Rolwes 2009; Yang and Du 2015). In this paper, we focus on the estimation of the expected credit loss for secured lending, which represents a significant part of the overall credit portfolio of a typical commercial bank.

In adapting the A-IRB models for IFRS 9 use, we need to be aware of a number of fundamental differences between the IFRS 9 and A-IRB parameters.

(1) A-IRB parameters are estimated based on a one-year risk horizon (as the Basel II capital horizon is one year), whereas IFRS 9 parameters need to be estimated in intervals till the maturity of each product (ie, the whole term structure is needed). This is because under IFRS 9 both one-year EL and "lifetime" EL (that is, the EL estimated over the effective maturity of the product) are required.

(2) The A-IRB PD parameter could be either conditional (typically known as point-in-time (PIT)) or unconditional (typically known as through-the-cycle (TTC)). The risk rating philosophy of a bank governs the choice between the two. Nevertheless, in most cases, banks use a hybrid PD philosophy with elements of both, but with a bias toward TTC (Miu and Ozdemir 2010). The A-IRB LGD and EaD parameters are typically TTC. Therefore, an EL estimate based on these A-IRB estimates would be a predominately TTC or "unconditional" estimate of the expected losses. IFRS 9, on the other hand, calls for a conditional EL that requires all PD, LGD and EaD parameters to be conditioned on the expected macroeconomic environment. Conditional PD estimations have

[3] McPhail and McPhail (2014) highlight the strengths and weaknesses of different modeling approaches that may be used to forecast credit losses.

commonly been used for stress testing under A-IRB. The use of conditional (ie, PIT) LGD and EaD estimates is, however, less common for A-IRB purposes (Ozdemir and Miu 2008).

(3) In general, A-IRB loss estimation is an "economic loss", whereas IFRS 9 loss estimation is an "accounting loss". There are several divergences in LGD estimations.

   (a) The down-turn LGD adjustment used in A-IRB is not appropriate for IFRS 9, as it is an adjustment for the tail of the loss distribution to compensate for the fact that the Basel II formula ignores the correlations between the PD and LGD.

   (b) Indirect recovery expenses (eg, overheads), which are included in A-IRB's economic loss by being incorporated into LGD estimations, are supposed to be excluded from the IFRS 9 calculation, as they cannot be allocated directly to individual loans from an accounting standpoint.

   (c) Another potential divergence will occur if IFRS 9's accounting LGD will not use the same discount rate for recovery cashflows as used by the A-IRB's economic LGD.

In this paper, we address different modeling issues related to (1) and (2) above. The "economic" versus "accounting" measurement issue in (3) is considered to be relatively less material, and we defer the detailed discussion of it to a future study.

This paper contributes to the practice of credit risk modeling by banks and financial institutions in adopting IFRS 9 by examining how we may use A-IRB models in the estimation of expected credit losses. In doing so, we highlight the importance of the necessary model adaptations required to satisfy the new accounting standard of impairment measurement. By leveraging on the A-IRB models, banks can lessen their modeling efforts in fulfilling IFRS 9, and capture the synergy between different modeling endeavors within the institutions. In outlining the proposed PD, LGD and EaD models, we provide detailed examples of how they may be implemented on secured lending. Moreover, in discussing the issues related to the estimation of the expected credit loss for IFRS 9, we highlight the challenges involved, and propose practical solutions to deal with them. For instance, one of the difficulties is in the calculation of the expected impairment loss through scenario analysis, which appears to be a common practice in forecasting the twelve-month or lifetime credit loss of an asset. Under this approach, we first estimate the losses under different plausible (economic) scenarios and then calculate the most likely value of the loss by evaluating a probability-weighted average value across the different scenarios. Assigning robust probabilities consistently to future scenarios is very difficult (if it is achievable at all).

The inherent subjectively will create material dispersion of results among financial institutions with similar underlying assets. It will therefore lead to difficulty in interpreting the results for the regulators and the investors alike. Dealing with multiple scenarios is also a considerable extra operational effort. In this paper, we propose a novel approach to deal with the issue without necessarily involving multiple scenarios, which can enhance the objectivity and replicability of the modeling results (see Section 2 for details).

We structure the paper as follows. In Section 2, we provide an overview of the task to calculate expected credit loss, and of the models that need to be developed under IFRS 9. We also introduce a mechanism to classify assets into the three risk buckets and discuss the implication of IFRS 9 on the procyclicality of credit provision. In Sections 3 and 4, respectively, we show how the conditional PD and LGD models for secured lending may be formulated based on their A-IRB counterparts. In Section 5, we examine the methodology for the estimation of EaD, together with the integration of the conditional parameters, to arrive at an EL measure for IFRS 9. We conclude with a few remarks in Section 6.

2 EXPECTED LOSS ASSESSMENT FRAMEWORK UNDER IFRS 9

2.1 Forward-looking expected loss impairment model

A key element of IFRS 9 is a forward-looking EL impairment model. The new standard requires that the EL estimate be forward-looking and incorporate available information at the time of estimation. Should a credit downturn (or upturn) be expected to "materially impact" the forward-looking credit quality of the obligors, "adjustments" to the (EL-based) estimates of the provisions are required.

IFRS 9 is not prescriptive about how exactly the changes in the credit/macroeconomic environment should be reflected in the EL estimation. However, a replicable, transparent and defendable mechanism to translate the change in the credit environment to the change in the portfolio's EL estimation is needed. Essentially, to satisfy IFRS 9, we are interested in calculating the EL of a credit facility, which can be defined as

$$\mathrm{EL} = E[\mathrm{LGD} \times \mathrm{EaD}] \times \mathrm{PD}, \tag{2.1}$$

where PD is probability of default, LGD is the random variable of loss given default and EaD is the random variable of exposure at default of the facility. The PD and the expectation are assessed based on the current forecast of the credit environment (CFCE). The definitions of PD, LGD and EaD under IFRS 9 are essentially the same as those under the Basel II A-IRB approach, perhaps with the minor exception that, in calculating LGD in IFRS 9, we need to exclude those indirect costs (eg, overheads) that cannot be attributed to individual facilities.

In practice, it is common to articulate the CFCE in terms of some kind of probability measure. Let us suppose the CFCE is characterized in terms of gross domestic product (GDP), and that we want to calculate the EL under the assumption that there are 35%, 50% and 15% chances that the GDP growth rate is 1.0%, 1.5% and 2.0%, respectively. Equation (2.1) can therefore be expressed as

$$\begin{aligned}\text{EL} = {} & E[\text{LGD} \times \text{EaD} \mid \text{GDP} = 1.0\%] \times \text{PD}(\text{GDP} = 1.0\%) \times 35\% \\ & + E[\text{LGD} \times \text{EaD} \mid \text{GDP} = 1.5\%] \times \text{PD}(\text{GDP} = 1.5\%) \times 50\% \\ & + E[\text{LGD} \times \text{EaD} \mid \text{GDP} = 2.0\%] \times \text{PD}(\text{GDP} = 2.0\%) \times 15\%, \end{aligned} \tag{2.2}$$

where $E[\text{LGD} \times \text{EaD} \mid \text{GDP} = X\%]$ denotes the expected value of LGD $\times$ EaD of the facility conditional on a GDP growth rate equal to $X\%$, and $\text{PD}(\text{GDP} = X\%)$ denotes the probability of default assessment of the facility conditional on a GDP growth rate equal to $X\%$. Equation (2.2) is therefore an EL assessment according to the "expected" economic outlook at the time of estimation. If we take the usual approximation by assuming LGD and EaD are independent, (2.2) becomes

$$\begin{aligned}\text{EL} \cong {} & E[\text{LGD} \mid \text{GDP} = 1.0\%] \times E[\text{EaD} \mid \text{GDP} = 1.0\%] \\ & \times \text{PD}(\text{GDP} = 1.0\%) \times 35\% \\ & + E[\text{LGD} \mid \text{GDP} = 1.5\%] \times E[\text{EaD} \mid \text{GDP} = 1.5\%] \\ & \quad \times \text{PD}(\text{GDP} = 1.5\%) \times 50\% \\ & + E[\text{LGD} \mid \text{GDP} = 2.0\%] \times E[\text{EaD} \mid \text{GDP} = 2.0\%] \\ & \quad \times \text{PD}(\text{GDP} = 2.0\%) \times 15\%. \end{aligned} \tag{2.3}$$

Banks have been conducting their stress testing exercise (eg, for Comprehensive Capital Analysis and Review (CCAR) and Internal Capital Adequacy Assessment Process (ICAAP) purposes under Basel II) by estimating conditional EL under a "stressed outlook" or, more precisely, under an economic outlook that corresponds to a selected "stress scenario"; eg, when the GDP growth rate equals -2.0%,

$$\begin{aligned}\text{EL} &= E[\text{LGD} \times \text{EaD} \mid \text{GDP} = -2.0\%] \times \text{PD}(\text{GDP} = -2.0\%) \\ &\cong E[\text{LGD} \mid \text{GDP} = -2.0\%] \times E[\text{EaD} \mid \text{GDP} = -2.0\%] \\ &\quad \times \text{PD}(\text{GDP} = -2.0\%). \end{aligned} \tag{2.4}$$

Thus, both IFRS 9 and stress tests are conditional estimates, but what they are conditioned on is very different. Although IFRS 9 is not a stress testing exercise, banks can utilize their existing PD stress testing models for IFRS 9 purposes as long as they use the current forecast, as opposed to the stress scenario, as their input. Let us consider how we can operationalize this. Suppose we have a PD stress testing model developed for A-IRB purposes that allows us to calculate the PD conditional on the GDP growth rate. Typically, such a model is forward-looking, relating the PD over the next year to today's observed information (ie, GDP), so that the PD can be used for the capital requirement calculation over a twelve-month risk horizon starting today.[4] Suppose the most recent GDP statistic (say GDP has grown by 1.7%) was published two months ago, and negative information on the economy has been revealed during the previous two months. Given that this negative information has not yet been captured by the most recent GDP statistics, we need to come up with a "forecast" on the GDP growth rate that we believe can more accurately reflect the current state of the credit environment. Suppose, based on our assessment, a lower GDP growth rate forecast of 1.5% is considered to be most probable.[5] We then use the GDP growth rate of 1.5% as our stress testing model input to calculate the PD, and in turn the EL, over the next twelve months for IFRS 9 purposes. Besides the EL over the next twelve months, IFRS 9 also calls for the calculation of "lifetime" EL (more details provided below). To satisfy this objective, we need to adapt our existing stress testing models so they can be used to calculate PD over any twelve-month period in the future. Suppose we want to calculate the EL over a twelve-month period starting three months from now. Given the current economic environment, we need to first come up with a forecast of the GDP growth rate over the twelve-month period ending three months from now. We then input it into the stress testing model to obtain a PD corresponding to that twelve-month period, enabling us to calculate the respective EL.

There is one more practical issue we need to deal with. Note that in (2.3) it might not be good enough to calculate EL by simply evaluating the product of PD and the expected values of LGD and EaD under the "expected" economic condition (ie, based on the expected GDP growth rate of $1.0\% \times 35\% + 1.5\% \times 50\% + 2.0\% \times 15\% = 1.4\%$). This is because PD(GDP), $E[\text{LGD} \mid \text{GDP}]$ and $E[\text{EaD} \mid \text{GDP}]$ are not necessarily linear functions of GDP, and thus

$$\begin{aligned}\text{PD}(\text{GDP} = 1.4\%) \neq \text{PD}(\text{GDP} = 1.0\%) &\times 35\% \\ + \text{PD}(\text{GDP} = 1.5\%) &\times 50\% \\ + \text{PD}(\text{GDP} = 2.0\%) &\times 15\%,\end{aligned} \tag{2.5a}$$

[4] It is quite likely that such a PD stress testing model is built on a statistical framework, where the point estimate of PD is subject to a certain degree of estimation errors. In our methodology, we abstract from such estimation errors.

[5] How do we come up with the 1.5% annual GDP growth rate "forecast" for today? One way is to first forecast the annual GDP growth rate to be realized at the next GDP publication date, which is ten months from now, given the current economic outlook. Suppose the forecast is 0.5%. We then interpolate 1.7% (the GDP growth rate two months ago) and 0.5% (the GDP growth rate ten months from now) over time to obtain a "forecast" of 1.5% for today.

$$
\begin{aligned}
E[\text{LGD} \mid \text{GDP} = 1.4\%] \neq E&[\text{LGD} \mid \text{GDP} = 1.0\%] \times 35\% \\
&+ E[\text{LGD} \mid \text{GDP} = 1.5\%] \times 50\% \\
&+ E[\text{LGD} \mid \text{GDP} = 2.0\%] \times 15\%, \qquad (2.5\text{b})
\end{aligned}
$$

$$
\begin{aligned}
E[\text{EaD} \mid \text{GDP} = 1.4\%] \neq E&[\text{EaD} \mid \text{GDP} = 1.0\%] \times 35\% \\
&+ E[\text{EaD} \mid \text{GDP} = 1.5\%] \times 50\% \\
&+ E[\text{EaD} \mid \text{GDP} = 2.0\%] \times 15\%. \qquad (2.5\text{c})
\end{aligned}
$$

In other words, specifying only the expected economic condition of GDP $= 1.4\%$ is not sufficient to calculate EL. The CFCE can only be fully defined, and thus the EL ascertained, by also specifying the probabilities (ie, 35%, 50% and 15%) of realizing all possible economic conditions (GDP $= 1.0\%$, 1.5% and 2.0%). It could be tricky even to assign probability measures to near-term economic conditions, let alone assigning those for economic conditions to be realized more than a couple of years from now. In practice, the assignment of probability measures is quite likely to be ad hoc and subjective. Such probability measures are also difficult to estimate in a consistent fashion. The decision process is therefore not easy to replicate, and thus the outcomes are not easily defendable. In the online appendix, we outline a methodology to correct for the bias in the EL calculation as a result of the nonlinearity in the functional forms of PD(GDP), $E[\text{LGD} \mid \text{GDP}]$ and $E[\text{EaD} \mid \text{GDP}]$. By doing so we can evaluate EL with only the point estimate of the expected economic condition and its standard deviation, which could be determined in a more objective way than the full probability measure.

2.2 EL buckets and their PD-based triggers

IFRS 9 requires the evaluation of different kinds of EL measures for loans classified into three different "buckets" of progressively higher loss potential. Specifically, the EL of loans in bucket 1 is estimated over a one-year horizon, whereas the EL of loans in bucket 2 is calculated over the remaining term to maturity (referred to as "life-time" EL). Finally, the EL of the impaired loans in bucket 3 is estimated based on the best estimates of recovery values.[6] IFRS 9 calls for a replicable, transparent and defendable mechanism to move loans among the three buckets with respect to significant changes in the expected credit environment. Loans will be reclassified from one bucket to another if certain predefined triggers are activated. It is important to note that IFRS 9 requires these triggers be based on PD only. Specifically, the trigger is based on the PD under the CFCE. In terms of our above example, the PD

[6] IFRS 9 also calls for lifetime EL for impaired loans in bucket 3. It can be argued that the lifetime of these impaired loans is shorter. However, a one-to-two-year workout period can be expected for some asset classes.

can be evaluated as

$$E[\text{PD} \mid \text{under CFCE}] = \text{PD}(\text{GDP} = 1.0\%) \times 35\% + \text{PD}(\text{GDP} = 1.5\%) \times 50\% + \text{PD}(\text{GDP} = 2.0\%) \times 15\%,$$

It is not ideal to ignore any changes in LGD and EaD in devising the triggers. Suppose the change in the economic condition underlying the CFCE affects only LGD (eg, the collateral value goes down materially) and/or EaD (eg, the utilized (drawn) amount increases materially), but not the PD of the facility. Based on a PD-based trigger, as required by IFRS 9, this facility will not be "downgraded" to a lower credit quality bucket (eg, from bucket 1 to bucket 2), even though the potential loss that could be incurred has increased, owing to the higher LGD and/or EaD. The lack of inclusion of LGD and EaD elements when setting the triggers hinders our ability to accurately capture the EL in a timely fashion.[7]

Having stated the shortcoming of having triggers that are solely based on PD, below we outline an approach to define these PD-based triggers based on the one-year conditional PD.[8]

(1) The trigger to move bucket 1 corporate obligors that originated as investment grade to bucket 2:[9] when the expected PD under the CFCE exceeds the investment grade PD threshold, the loan can no longer be considered as investment grade and must be moved from bucket 1 to bucket 2, where lifetime EL applies. That is, when

$$E[\text{PD} \mid \text{under CFCE}] > \text{investment grade PD level threshold.}$$

The investment grade PD threshold is therefore the maximum conditional (PIT) PD allowed for an investment grade obligor. Note that some obligors will remain

[7] Note that the increase in LGD and/or EaD is only captured when and if EL is recalculated.

[8] Alternatively, the triggers can be set based on lifetime (cumulative) PD. This, however, creates undesirable complexity in practice for two reasons. First, the estimation of forward PD requires the capability of reliably forecasting the term structure of CFCE, which is difficult. Second, since lifetime PD is a function of the remaining time to maturity of the asset, any change in the lifetime PD could be the results of either a change in the credit quality of the borrower or simply the (natural) shortening of the time to maturity over time, or both. If an increase in lifetime PD is indeed due to the deterioration of credit quality, it warrants moving the asset to a lower bucket. However, it would be unfair to assign the asset to a higher-risk bucket if the increase in lifetime PD is simply the result of the latter. In practice, it is difficult to disentangle these two confounding effects. Given these complexities and potential estimation errors, here, we present the triggers based on the one-year conditional PD.

[9] The IFRS 9 rule only refers to "high-quality credit", which is not necessarily "investment grade". Therefore, in practice, each bank will need to come up with its own definition of high-quality credit and the corresponding bucketing rule. In the proposed bucketing rule, we are defining high-quality credit as investment-grade obligors.

in bucket 1 despite the material increase in their PD, as long as they remain at investment grade. This can be justified from the standpoint that, despite the increase in EL due to an increase in PD, the absolute value of the PD remains low enough (as investment grade) not to warrant a migration to bucket 2.

This can work in reverse: the obligors move from bucket 2 back to bucket 1 when the credit quality improves, satisfying

$$E[\text{PD} \mid \text{under CFCE}] \leqslant \text{investment grade PD level threshold.}$$

(2) The trigger to move bucket 1 corporate obligors that originated as noninvestment grade to bucket 2: as these obligors originated as noninvestment grade, their current PDs are likely to be higher than the investment grade conditional PD threshold mentioned above. To define the trigger for moving from bucket 1 to 2, we therefore instead use a proportional threshold that is a benchmark against which to test credit quality at origination:

$$\frac{E[\text{PD} \mid \text{CFCE}] - E[\text{PD} \mid \text{FCE}^{\text{at origination}}]}{E[\text{PD} \mid \text{FCE}^{\text{at origination}}]} > \%\ \text{threshold}, \tag{2.6}$$

where $E[\text{PD} \mid \text{FCE}^{\text{at origination}}]$ is the expected PD given the forecast of the credit environment at the origination of the loan ($\text{FCE}^{\text{at origination}}$). The proportional threshold (% threshold) is chosen to reflect a material enough relative increase in the conditional PD (eg, 10%). This trigger can also work in reverse, and the obligor moves from bucket 2 back to bucket 1 when its credit quality improves, satisfying

$$\frac{E[\text{PD} \mid \text{CFCE}] - E[\text{PD} \mid \text{FCE}^{\text{at origination}}]}{E[\text{PD} \mid \text{FCE}^{\text{at origination}}]} \leqslant \%\ \text{threshold}.$$

(3) The trigger to move retail exposures from bucket 1 to bucket 2: typically retail obligors are pooled, and therefore are not classified into investment versus noninvestment grades. The above triggers for corporate obligors are therefore not applicable to retail exposures. We should nevertheless follow the same principle. In order to ensure the movement between buckets will only be triggered by a material change in EL, the thresholds need to be dependent on the PD level. That is, if the PD is sufficiently low to begin with, its increasing by multiples should not necessarily warrant moving the asset from bucket 1 to 2, as the absolute level of PD is still considered to be low after the increase. For example, an increase from a PD of 0.15% to 0.45% – a threefold increase – may not be enough to warrant a downgrade; whereas an increase from a PD of 5.00% to

7.50% – only a 50% increase – may do so. This could be accommodated by imposing a double-trigger requirement:

$$E[\text{PD} \mid \text{under CFCE}] > \text{PD level threshold}$$

and

$$\frac{E[\text{PD} \mid \text{CFCE}] - E[\text{PD} \mid \text{FCE}^{\text{at origination}}]}{E[\text{PD} \mid \text{FCE}^{\text{at origination}}]} > \%\ \text{threshold.} \tag{2.7}$$

The (double) trigger also works in reverse. The asset will be moved from bucket 2 back to bucket 1 when the credit quality improves, satisfying

$$E[\text{PD} \mid \text{under CFCE}] \leqslant \text{PD level threshold}$$

or

$$\frac{E[\text{PD} \mid \text{CFCE}] - E[\text{PD} \mid \text{FCE}^{\text{at origination}}]}{E[\text{PD} \mid \text{FCE}^{\text{at origination}}]} \leqslant \%\ \text{threshold.}$$

(4) The trigger to move from buckets 1 and 2 to bucket 3: nonperforming obligors will be moved from buckets 1 and 2 to bucket 3 when their conditional PDs exceed the “performing” grade PD threshold, that is, when

$$E[\text{PD} \mid \text{under CFCE}] > \text{performing grade PD threshold.} \tag{2.8}$$

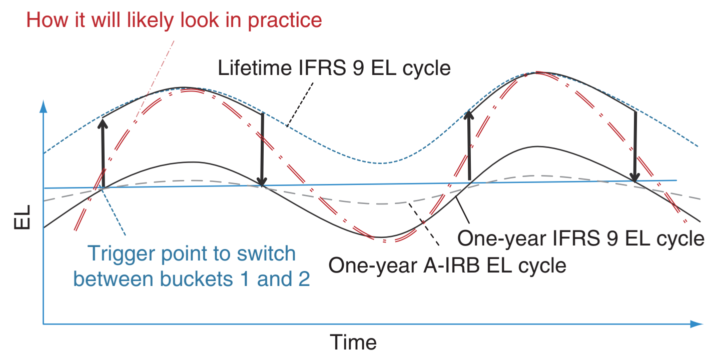
The performing grade PD threshold is likely to be above 50%, and needs to be defined in the institution’s internal policies.

To ensure that we are able to capture both quantitative and qualitative information in a timely fashion, the mechanical thresholds discussed above should be supplemented with expert-judgment-based thresholds devised based on a bank’s experience of its specific credit portfolios. For example, besides judging based on the conditional PD of the obligor, the bank may have other information that will suggest a loan should be considered as nonperforming (eg, the loan is already ninety days past due).

2.3 Procyclicality in provision requirement under IFRS 9

There are two sources of procyclicality in credit provision introduced by IFRS 9. First, the one-year EL measure conditional on CFCE is deemed to be procyclical, as all of its components, namely PD, expected LGD and expected EaD, are functions of the prevailing economic condition and thus PIT in nature. This EL measure under IFRS 9 is expected to be more procyclical than the EL based on A-IRB parameters. This is because, in practice, the PD parameter under A-IRB is likely to be less PIT than that under IFRS 9, not to mention the LGD and EaD parameters under A-IRB

FIGURE 1 Adoption of IFRS 9 will amplify the procyclicality of credit provision.



are typically TTC. Second, the procyclicality will be amplified as obligors switch from bucket 1 to bucket 2 during a downturn, while switching from bucket 2 back to bucket 1 as the credit condition improves. Figure 1 illustrates how EL will behave over two business cycles under IFRS 9. Suppose initially all obligors are in bucket 1. When the credit condition starts to degenerate, the one-year EL increases given the higher estimations of PD and expected LGD and EaD. As the credit condition continues to degenerate, the PD-based threshold is breached for (at least some of) the obligors. These obligors are thus moved to bucket 2. The switching from a one-year EL to a lifetime EL measure by itself will result in a higher overall EL, thus amplifying the effect of the worsening market condition. As the credit condition finally starts to improve and eventually (perhaps some of) the bucket 2 obligors' PD becomes lower than the respective thresholds, they are moved from bucket 2 back to bucket 1. Switching from a lifetime EL back to a one-year EL measure will result in a significant drop in the overall EL, again amplifying the sensitivity of EL to the changing business cycle conditions. It is important to note that the amplification as a result of switching between buckets is directly proportional to the duration of the portfolio. The longer the duration, the greater the difference between one-year EL and lifetime EL, and the more procyclicality there will be.

A natural extension to this discussion is the increased disconnect between the capital requirement (under Basel II) and the credit provisions (under IFRS 9). Note that the capital requirement is defined net of the one-year EL under both A-IRB's regulatory and economic capital frameworks. The fact that IFRS 9's one-year EL is more procyclical than A-IRB's (always one-year) EL introduces the first disconnect between the two measures. The more significant issue, however, is the switching between one-year EL and lifetime EL under IFRS 9. When measuring the provision based on lifetime EL while the capital requirement is net of the one-year EL, we are indeed double counting the credit loss by the difference between the one-year EL and lifetime EL. This is an issue arising from the disconnect between the risk horizons assumed in the capital requirement and reserve estimations. While the former is always one year, the latter may switch to lifetime. It may be argued that, although the prescribed risk horizon is one year under the Basel II Pillar 1 regulatory capital requirement, banks do assess their capital adequacy over a longer horizon in conducting their ICAAP and/or CCAR. We understand at the time of writing that when IFRS 9-based reserves exceed the one-year EL under A-IRB, banks will be able to recognize some Tier 1 capital benefit up to a certain threshold. Nevertheless, any remaining double counting or disconnect between the capital requirement and reserves requirement will add further complexity and potential surprises to the banks' management of their reserve and capital levels.

3 ESTIMATION OF PROBABILITY OF DEFAULT

In this section, we examine how we may adapt the stress testing models commonly used in fulfilling Basel II A-IRB requirements for calculating conditional PD under IFRS 9. The so-called top-down PD stress testing models exploit the statistical relation between systematic PD implicit in a credit portfolio and macroeconomic variables that govern the underlying credit risk. For example, building on the single-factor infinitely granular portfolio credit risk model of Vasicek (1987), Miu and Ozdemir (2009) proposed a time series model to calculate risk-rating-specific PD under predefined stressed scenarios that could be articulated with observable macroeconomic variables. Besides being used to generate stressed PD, the model can also be used to calculate PD conditional on the current or expected outlook of the economy. Thus, it not only serves as a tool for assessing risk capital requirement under A-IRB, but can also be used to calculate forward-looking conditional PD in satisfying IFRS 9. In the following, we outline the conditional PD model adapted from Miu and Ozdemir (2009) and demonstrate the implementation of such a model in assessing the conditional PD for a representative residential mortgage portfolio.

Suppose borrowers are uniform in terms of their credit risks within a certain segment (eg, a specific risk rating) of the portfolio. Their individual PD risk p_t at time t is driven by both the systematic PD risk P_t and the borrower-specific PD risk e_t.[10] For example, for borrower i,

$$p_t^i = R \times P_t + \sqrt{1 - R^2} \times e_t^i. \tag{3.1}$$

[10] This approach of decomposing an obligor's default risk into its systematic and idiosyncratic components is consistent with the model underlying the Basel II, Pillar 1 risk-weight function.

Both P_t and $e_{\mathrm{PD},t}$ are assumed to follow the standard normal distribution. Thus, p_t also follows the standard normal distribution. Under Merton's framework (Merton 1973), we can interpret p_t as a latent variable, which is a normalized function of the borrower's asset return. For retail credit facilities, we may interpret p_t as a normalized measure of the financial health of the individual borrower, which varies with both systematic factors and borrower-specific conditions. The borrower defaults on their loan when p_t becomes less than some constant default point (DP). Thus, the smaller the value of p_t (ie, the closer to DP), the greater the borrower's PD. The coefficient R is assumed to be uniform across borrowers and measures the sensitivity of individual risks to the systematic PD risk, P_t. The parameter R^2 is therefore the pairwise correlation of p_t between borrowers as a result of the systematic risk factor. Equation (3.1) is in fact the single-factor model considered by Vasicek (1987) in deriving the loss distribution of a credit portfolio.[11]

We can define the long-run probability of default (LRPD) of a particular risk rating m (where $m = 1, 2, \dots, M$) as the unconditional probability of p_t being lower than the risk-rating-specific default point (DP_m):[12]

$$\mathrm{LRPD}_m = \Pr[p_t^i < \mathrm{DP}_m]. \tag{3.2}$$

LRPD is therefore a function of DP_m. This function is defined by the unconditional distribution of p_t, which is in turn governed by the unconditional distribution of P_t via (3.1). To allow for the computation of conditional PD, we can model the systematic PD risk P_t as a function $f(\cdot)$ of, say, J explanatory variables $X_t^1, X_t^2, \dots, X_t^J$:

$$P_t = f(X_t^1, X_t^2, \dots, X_t^J, X_{t-1}^1, X_{t-1}^2, \dots, X_{t-1}^J, \dots) + \varepsilon_t. \tag{3.3}$$

These explanatory variables could be macroeconomic variables, market variables and/or economic indicators that are expected to be able to explain the systematic PD risk

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[11] The single-factor Vasicek model under Merton's framework is arguably more relevant for modeling the credit risk of wholesale portfolios than that of retail portfolios, given that the notion of "asset return" is more appropriate for obligors in the former than in the latter. Nevertheless, it is a commonly adopted approach in modeling the PD of both kinds of portfolios. For example, the calculations of Basel II, Pillar 1 risk-weighted assets for both wholesale and retail portfolios are formulated based on the single-factor Vasicek model. It is also important to emphasize that we are not assuming different kinds of retail portfolios (eg, mortgages, credit cards, etc) are driven by the same single risk factor. Separate conditional PD models are constructed for different kinds of retail portfolios, each driven by their own specific risk factor. Finally, the pairwise correlation R_2 is also portfolio specific. For example, wholesale portfolios tend to have a higher pairwise correlation than retail portfolios. In the calibration of long-run probability of default (LRPD) and the generation of conditional PD, we need to estimate and use the appropriate pairwise correlation for the specific portfolio under consideration.

[12] Note that superscript i denotes the ith borrower within the uniform risk rating.

of the credit portfolio under consideration. Some examples of explanatory variables are: real GDP growth rate; unemployment rate; house price index; interest rate; stock market index return. Note that, in (3.3), besides the contemporaneous values of the explanatory variables, we may also include lagged values of these variables as possible additional explanatory variables. The first term of (3.3) (ie, $f(\cdot)$) may be interpreted as the explainable component of P_t, while the second term (ie, the residual term ε_t) is the unexplainable component. Under this framework LRPD, R^2 and the other parameters governing function $f(\cdot)$ can be estimated by observing the time series of historical default rates and explanatory variables (for details, see Miu and Ozdemir 2009).[13]

After calibrating the model, we can then use it to generate risk-rating-specific PD conditional on the current and expected economic outlooks so as to satisfy IFRS 9. Specifically, the term structure of the forward one-year conditional PD over the first, second and third years (PD_1, PD_2 and PD_3, respectively) can be calculated by evaluating the following conditional probabilities according to the term profile of the expected values x of the explanatory variables X. For example, for risk rating m,

$$\text{PD}_{m,1} = \Pr[p_1^i < \text{DP}_m \mid X_1^1 = x_1^1, X_1^2 = x_1^2, \dots, X_1^J = x_1^J], \tag{3.4a}$$

$$\text{PD}_{m,2} = \Pr[p_2^i < \text{DP}_m \mid X_2^1 = x_2^1, X_2^2 = x_2^2, \dots, X_2^J = x_2^J], \tag{3.4b}$$

$$\text{PD}_{m,3} = \Pr[p_3^i < \text{DP}_m \mid X_3^1 = x_3^1, X_3^2 = x_3^2, \dots, X_3^J = x_3^J]. \tag{3.4c}$$

We now provide an example of implementing the above conditional PD model on a representative residential mortgage portfolio. Similar to many other credit portfolios, we do not have a sufficiently long historical default rate data series for this mortgage portfolio to calibrate the above model robustly. An external proxy is therefore used in the selection of explanatory variables and the calibration of the parameters of the function $f(\cdot)$.[14] We use the publicly available proportion of mortgages in arrears at the national level as our default rate proxy. We thus assume the same set of explanatory variables is driving both the proportion of mortgages in arrears and the default rate of our mortgage portfolio. The model development involves the following two-step process.

[13] By formulating the systematic PD risk P_t as a function of multiple explanatory variables in (3.3), we are in effect extending the single-factor representation of default risk to a multifactor one. This multifactor representation allows for more flexibility in modeling the conditional PD for different kinds of credit portfolios.

[14] The use of an external proxy may be challenged by model validation on data representativeness and key assumption risk. Therefore, it is important that representativeness of the external proxy is justified and a robust methodology is employed to explicitly account for the differences in asset correlations and the level of PDs between the internal portfolio and the external proxy (see Miu and Ozdemir (2008) for an example of such a methodology).

TABLE 1 Estimated coefficients of the best performing model.

Variable	Parameter	Point estimate
Intercept	a	–17.441
Annual growth rate of real GDP	b_1	0.113
Annual change of unemployment rate	b_2	–0.150
Quarterly growth rate of residential property resale price	b_3	0.050
Detrended value of number of housing units started	b_4	0.111
Quarterly change in five-year residential mortgage rate	b_5	0.022

Step 1: using the historical data of our default risk proxy, we compare the explanatory power of different specifications of $f(\cdot)$ by considering different combinations of potential explanatory variables. Essentially, we assume the systematic PD risk, P_t, implicit in the external proxy is identical to that of our mortgage portfolio. We conduct our estimations using both single-variable and multiple-variable specifications of $f(\cdot)$ in order to identify the specification that is both intuitive and of the highest explanatory power.

Step 2: we estimate the default point, DP_m, using the (limited) internal default rate data of our mortgage portfolio. Here, we assume the function $f(\cdot)$ of the "best-fitted" model identified and calibrated in step 1 is equally applicable to our mortgage portfolio. An appropriate value of R^2 is also selected so that the conditional PD outputs of the calibrated model match the observed default rates of our mortgage portfolio as close as possible.

A total of twenty-eight variables are considered in step 1 in testing their explanatory power on the systematic PD risk implicit in the proxy. They include general economic variables (eg, real GDP, unemployment rate), various interest rates, exchange rate, stock market index, confidence index and various housing market data, and are considered to be key drivers/indicators of mortgage credit risks.

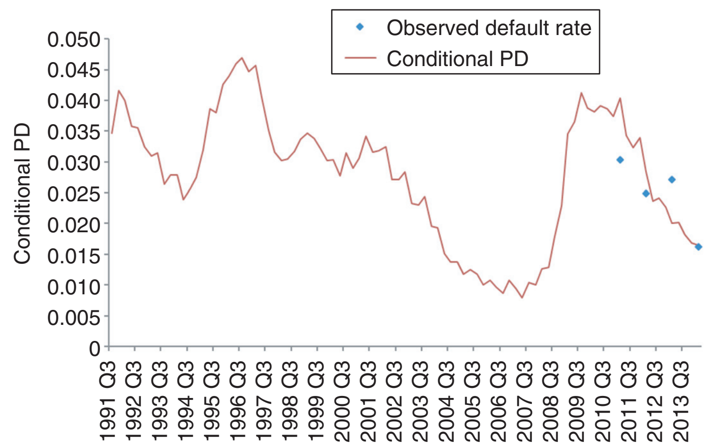
We consider a sample period from 1986 to 2014. For each of these variables, we consider different versions of the time series, including its level, annual change, quarterly change, annual proportional change, quarterly proportional change and its detrended value.[15]

We specify a linear representation of the function $f(\cdot)$:

$$f(\cdot) = a + b_1 X_t^1 + \cdots + b_J X_t^J \tag{3.5}$$

[15] Standard stationary tests are conducted on the variables to ensure nonstationary time series will be excluded from the modeling exercise.

FIGURE 2 Conditional PD based on the calibrated model, and realized default rates at portfolio level from 2010 to 2013.



and measure model performance based on in-sample goodness-of-fit over our sample period. The best performing model is made up of five explanatory variables: annual growth rate of real GDP; annual change of unemployment rate; quarterly growth rate of residential property resale price; detrended value of the number of housing units started; and quarterly change in five-year residential mortgage rate. The estimated coefficient values are reported in Table 1.

After calibrating the time series model, we then calculate the DP and LRPD for each of the risk ratings of the mortgage portfolio. Finally, the one-year PD of each risk rating can be calculated conditional on the realization of the five underlying explanatory variables at different points in time. In Figure 2, we plot the conditional one-year PD of the aggregated mortgage portfolio (ie, across all risk ratings) based on actual values of the five variables realized from 1991 to 2014. For comparison, we also plot the realized default rates at the portfolio level from 2010 to 2013.

4 ESTIMATION OF LOSS GIVEN DEFAULT

In this section, we present a methodology to predict the "term structure" of LGD of secured facilities by modeling the "term structure" of the value of its underlying collateral. Collateral value is one of the most significant drivers of the ultimate recovery value (and thus the LGD) of a defaulted secured instrument. For example, the recovery value from a defaulted residential mortgage should be closely related to the value of the foreclosed housing property that will be disposed of during the workout process. The same can be said for a defaulted equipment financing contract, of which the recovery value is governed by the residual value (net of any depreciation as a result of wear and tear) of the equipment itself. Thus, if we can accurately forecast the value of the collateral, we will also be able to project the term structure of the LGD of the secured facility for the calculation of EL under IFRS 9.[16]

Before we introduce our model, let us start by stating some basic definitions and notation. Suppose T_{default} denotes the time of default of a secured facility. The percentage and dollar amount of LGD can be defined as

$$\text{LGD}_{T_{\text{default}}}\% = \left(1 - \frac{\text{PV}_{T_{\text{default}}}(\text{net recoveries})}{\text{EaD}_{T_{\text{default}}}}\right) \tag{4.1}$$

and

$$\text{LGD}_{T_{\text{default}}}\$ = \text{LGD}_{T_{\text{default}}}\% \times \text{EaD}_{T_{\text{default}}}, \tag{4.2}$$

where $\text{EaD}_{T_{\text{default}}}$ is the EaD of the facility as measured at default and the present value of recovery is the sum of net cashflows received during the workout process capitalized at an appropriate discount rate r. That is,

$$\text{PV}_{T_{\text{default}}}(\text{net recoveries}) = \sum_{m=1}^{M} (\text{recovery}_{t_m} - \text{cost}_{t_m}) \times \mathrm{e}^{-r\times(t_m - T_{\text{default}})}, \tag{4.3}$$

where there are M cashflows occurring at time $t_1, t_2, \dots, t_M$ with $T_{\text{default}} \leqslant t_1 \leqslant t_2 \leqslant \cdots \leqslant t_M$.

The present value of the recovery value is expected to be quite similar to the value of the underlying collateral measured at the time of default ($V_{T_{\text{default}}}$). Their difference is mainly attributable to

- the costs incurred in the recovery process (eg, legal fees, collection fees),
- the errors in the estimation of the collateral value at the time of default (eg, the appraisal value of a residential property at the time of default may be biased, the amortized value of leased equipment after depreciation may differ from its actual economic value).

Both the costs of recovery and the errors in collateral valuation are expected to be relatively insensitive to the prevailing state of the business cycle. One of the main assumptions of the proposed model is that the present value of recovery is assumed to be a fixed fraction (δ) of the value of the underlying collateral at the time of default ($V_{T_{\text{default}}}$). For a facility collateralized on a single asset, we therefore have

$$\text{PV}_{T_{\text{default}}}(\text{net recoveries}) = \delta \times V_{T_{\text{default}}}. \tag{4.4}$$

[16] To forecast the LGD of unsecured facilities, one may wish to refer to the conditional LGD models examined by Ozdemir and Miu (2008).

We can refer to δ as the "net recovery ratio", representing the proportion of collateral value at default that is recouped from the write-off/recovery process. We assume the value of δ is uniform across the same type of secured facilities (eg, a certain segment of residential mortgages) and thus can be estimated by taking the average of the ratios $\text{PV}_{T_{\text{default}}}(\text{net recoveries})/V_{T_{\text{default}}}$ for all defaulted loans within that specific secured facility type over a certain historical sample period. By specifying the recovery value as a fixed fraction of the collateral value, we can then predict the recovery value (and thus the LGD) by explicitly modeling the variation of the asset values underlying the collateral as functions of observable macroeconomic factors (eg, house price index, equipment price index).[17] Note that we can readily extend the above specification to cover those facilities against which multiple collaterals are pledged. For example, the recovery value of a loan collateralized on two different assets can be expressed as

$$\text{PV}_{T_{\text{default}}}(\text{net recoveries}) = \delta_1 \times V_{1,T_{\text{default}}} + \delta_2 \times V_{2,T_{\text{default}}}, \tag{4.5}$$

where $V_{1,T_{\text{default}}}$ and $V_{2,T_{\text{default}}}$ are the collateral values of the two assets at the time of default, and δ_1 and δ_2 are the respective net recovery ratios applicable to the two kinds of assets.

In order to predict the recovery value conditional on the current and/or expected economic outlooks, we propose the following regression model for the growth rate of the collateral value. Suppose today is time t_0 and we want to predict LGD for a default that will occur at future time $t_0 + \tau$. Let $r^V_{t_0,t_0+\tau}$ be the annualized growth rate of the collateral value from time t_0 to $t_0 + \tau$. Thus,

$$V_{t_0+\tau} = V_{t_0} \times \exp(\tau r^V_{t_0,t_0+\tau}). \tag{4.6}$$

The expected economic outlook is supposed to dictate the expected changes in collateral value and, in turn, its growth rate ($r^V_{t_0,t_0+\tau}$). For example, if we think the general residential property price (as measured by a certain house price index) will increase by 5% in the next twelve months, it is not unreasonable to expect that the collateral value of the residential property underlying a mortgage will tend to increase at a similar rate. On the other hand, depreciation (eg, in the form of wear and tear for leased equipment) will tend to result in a "negative drag" on the growth rate of collateral value. Finally, the appraised value of the collateral could be subject to different kinds of errors and/or biases when the loan is still performing versus when it has defaulted. Since, in applying (4.6), we are in fact predicting $V_{t_0+\tau}$ and interpreting it as $V_{T_{\text{default}}}$ (ie, $T_{\text{default}} = t_0 + \tau$) when using it in (4.4) to calculate the recovery value for a default that will occur at time $t_0 + \tau$, we also need to incorporate such differences

[17] While the assumption of recovery value as a fixed fraction of the collateral value is considered to be a practical solution, it should be stated that the ratio of recovery value to collateral value is likely to be sensitive to market supply and demand, which may vary with macroeconomic conditions.

in appraisal errors/biases in the change in collateral values. To incorporate the above determinants of the growth rate of the collateral value, we specify the following linear representation for $r^V_{t_0,t_0+\tau}$:

$$r^V_{t_0,t_0+\tau} = \alpha + \beta_1 r^{\text{factor}_1}_{t_0,t_0+\tau} + \beta_2 r^{\text{factor}_2}_{t_0,t_0+\tau} + \cdots + \beta_J r^{\text{factor}_J}_{t_0,t_0+\tau} + \epsilon_{t_0}, \tag{4.7}$$

where $r^{\text{factor}_1}_{t_0,t_0+\tau}, r^{\text{factor}_2}_{t_0,t_0+\tau}, \dots, r^{\text{factor}_J}_{t_0,t_0+\tau}$ are the annualized rates of changes of possibly J underlying drivers of the collateral value (eg, for residential mortgages, these factors may simply be different residential property resale price indexes; for equipment leasing, these factors may be different new and/or resale equipment price indexes). The coefficients $\beta_1, \beta_2, \dots, \beta_J$ measure the sensitivities of the collateral value of the specific asset under consideration to the different factors/indexes. Any depreciation will be represented by a negative intercept α in (4.7).[18] The same intercept will also capture any bias in the appraised value that is independent of any of the factors. If the bias in the appraised value is somehow a function of the factors (eg, the appraised value tends to be too optimistic (pessimistic) when the market condition tends to be good (bad)), it will also show up in the coefficients $\beta_1, \beta_2, \dots, \beta_J$. In practice, we will be working with the expected version of (4.7) and thus ignoring the random errors ϵ_{t_0}:

$$E_{t_0}[r^V_{t_0,t_0+\tau}] = \alpha + \beta_1 E_{t_0}[r^{\text{factor}_1}_{t_0,t_0+\tau}] + \beta_2 E_{t_0}[r^{\text{factor}_2}_{t_0,t_0+\tau}] + \cdots + \beta_J E_{t_0}[r^{\text{factor}_J}_{t_0,t_0+\tau}]. \tag{4.8}$$

Thus, according to (4.6), our prediction of the collateral value at the time of default, $T_{\text{default}} = t_0 + \tau$, is

$$E_{t_0}[V_{T_{\text{default}}}] = E_{t_0}[V_{t_0+\tau}] = V_{t_0} \times \exp(\tau E_{t_0}[r^V_{t_0,t_0+\tau}]). \tag{4.9}$$

As stated, this approach for collateral value growth rate is particularly suitable for mortgages or secured lending with specific collaterals whereby there is a tangible way of estimating the collateral value. This approach would be less suitable for blanket lien types of collateral. Then, according to (4.4), the expected recovery value at default can be expressed as

$$E_{t_0}[\text{PV}_{T_{\text{default}}=t_0+\tau}(\text{net recoveries})] = \delta \times V_{t_0} \times \exp(\tau E_{t_0}[r^V_{t_0,t_0+\tau}]). \tag{4.10}$$

[18] Without loss of generality, in specifying (4.7), we are assuming the intercept α, which captures the rate of depreciation and the bias in appraised value, is a constant with respect to the prediction horizon τ. We can easily generalize the model by specifying α to be a function of τ. This will cater for the situation in which the rate of depreciation is expected to be changing over the term of the contract, according to prior experience with that particular kind of asset.

Finally, according to (4.1), the conditional expected LGD for a default time that is τ periods from today is

$$
\begin{aligned}
&E_{t_0}[\mathrm{LGD}_{T_{\mathrm{default}}=t_0+\tau}\%]\\
&\quad=\left(1-\frac{\delta V_{t_0}\exp(\tau E_{t_0}[r^{V}_{t_0,t_0+\tau}])}{E_{t_0}[\mathrm{EaD}_{T_{\mathrm{default}}=t_0+\tau}]}\right)\\
&\quad=\left(1-\frac{\delta V_{t_0}\exp(\tau\{\alpha+\beta_1 E_{t_0}[r^{\mathrm{factor}_1}_{t_0,t_0+\tau}]+\beta_2 E_{t_0}[r^{\mathrm{factor}_2}_{t_0,t_0+\tau}]+\cdots+\beta_J E_{t_0}[r^{\mathrm{factor}_J}_{t_0,t_0+\tau}]\})}{E_{t_0}[\mathrm{EaD}_{T_{\mathrm{default}}=t_0+\tau}]}\right),
\end{aligned}
\tag{4.11}
$$

where $E_{t_0}[\mathrm{EaD}_{T_{\mathrm{default}}=t_0+\tau}]$ is the expected value of the EaD of the facility at time $t_0 + \tau$, which we discuss in detail in Section 5. By substituting $\tau = 1$ year, 2 years, $\dots$, N years into (4.11), we can then generate the forward-looking term structure of the LGD. Note that, besides knowing today's collateral value (V_{t_0}) and the expected EaD ($E_{t_0}[\mathrm{EaD}_{T_{\mathrm{default}}=t_0+\tau}]$) of the facility, to evaluate (4.11) we also need to specify the expected changes in values of the underlying factors (ie, $E_{t_0}[r^{\mathrm{factor}_1}_{t_0,t_0+\tau}]$, $E_{t_0}[r^{\mathrm{factor}_2}_{t_0,t_0+\tau}], \dots, E_{t_0}[r^{\mathrm{factor}_J}_{t_0,t_0+\tau}]$) that are consistent with the current market outlook under CFCE. More importantly, we need to find out the values of the model parameters δ, α and $\beta_1, \beta_2, \dots, \beta_J$. As mentioned above, the net recovery ratio, δ, is assumed to be a constant for the same types of secured facilities and thus can be estimated by taking the average of the ratios $\mathrm{PV}_{T_{\mathrm{default}}}(\text{net recoveries})/V_{T_{\mathrm{default}}}$ for all defaulted loans within that specific secured facility type over a certain historical sample period. The other parameters, α and $\beta_1, \beta_2, \dots, \beta_J$, are also supposed to be facility-type specific and can be estimated by conducting a regression analysis based on (4.7). Specifically, using historical collateral value data and the specified factors, we will regress the changes in collateral value for a certain type of facility (eg, within a segment of residential mortgages) against the contemporaneous changes in the factors of interest over a certain historical sample period. Rather than this being a purely empirical exercise, we may also want to impose some structure if some of the components that dictate the changes in collateral value are expected to behave in a deterministic fashion. For example, in equipment financing/leasing, where the equipment itself is the collateral, the equipment's rate of amortization implicit in its prespecified amortization schedule can serve as a reference in determining the value of the intercept term, α.

Let us end this section with a simple numerical example. Suppose we want to calculate the LGD to be applied to a default occurring exactly one year from today. Today's collateral value of a secured facility is \$100. Suppose there is only a single underlying factor driving the collateral value of facilities belonging to this specific asset class and the estimated values of δ, α and β for this asset class are 0.90, −0.30

TABLE 2 Conditional LGDs under different factor value scenarios.

	$E_{t_0}[r^{\text{factor}}_{t_0,t_0+1}]$ (%)		
	–10.0	**0.0**	**10.0**
$E_{t_0}[V_{t_0+1}]$ (\$)	68.0	74.1	80.7
$E_{t_0}[LGD_{t_0+1}\%]$ (%)	18.3	11.1	3.2

and 0.85, respectively. Based on an expected EaD of \$75 at year end, the conditional LGDs under different expected changes in the factor value (ie, $E_{t_0}[r^{\text{factor}}_{t_0,t_0+\tau}]$) are reported in Table 2.

5 ESTIMATION OF EXPOSURE AT DEFAULT AND EXPECTED LOSS

After calculating the conditional PD and the expected LGD, the last step is to estimate EL by incorporating information regarding the expected EaD of the facility. As discussed in Section 2, if we take the usual approximation by assuming LGD and EaD are independent, we have a one-year EL given by

$$\mathrm{EL}_{\text{1-year}} = E_{t_0}[\mathrm{LGD}_{t_0+1}] \times E_{t_0}[\mathrm{EaD}_{t_0+1}] \times \mathrm{PD}_1, \tag{5.1}$$

where $E_{t_0}[\mathrm{LGD}_{t_0+1}]$ and $E_{t_0}[\mathrm{EaD}_{t_0+1}]$ are the expected LGD and EaD of the facility conditional on CFCE if it defaults in the first year, and PD_1 is the probability of default of the obligor in the first year conditional on CFCE. We will be calculating this one-year EL for the exposures in bucket 1. For those in bucket 2, a lifetime EL is called for. The lifetime EL of a facility that lasts for n periods can be estimated by

$$\begin{aligned}\mathrm{EL}_{\text{lifetime}} = {} & E_{t_0}[\mathrm{LGD}_{t_0+1}] \times E_{t_0}[\mathrm{EaD}_{t_0+1}] \times \mathrm{PD}_1 \\ & + \sum_{\tau=2}^{n} E_{t_0}[\mathrm{LGD}_{t_0+\tau}] \times E_{t_0}[\mathrm{EaD}_{t_0+\tau}] \times \mathrm{PD}_\tau \prod_{j=1}^{\tau-1}(1-\mathrm{PD}_j),\end{aligned} \tag{5.2}$$

where $E_{t_0}[\mathrm{LGD}_{t_0+\tau}]$ and $E_{t_0}[\mathrm{EaD}_{t_0+\tau}]$ are the expected LGD and EaD of the facility conditional on CFCE if it defaults in the τth period from today, ie, t_0, and PD_τ is the one-period conditional probability of default of the obligor over the τth period that is consistent with CFCE. Note that the specification in (5.2) is flexible enough to cater to a lifetime EL calculation under a discrete risk assessment frequency that is other than one year. If LGD and EaD are not expected to be constant over time, the higher the frequency, the more accurate the EL estimation will be. However, in practice, it is quite unlikely that it will be more frequent than quarterly.

With the conditional PD and LGD models presented in Sections 3 and 4, we can estimate the one-year and lifetime EL based on (5.1) and (5.2) once we are able

TABLE 3 Conditional term structure of expected loss given default and exposure at default.

(a) Conditional term structure of expected LGD and EaD

	Year 1	Year 2	Year 3
τ	1	2	3
$E_{t_0}[r^{\text{factor}}_{t_0,t_0+\tau}]$ (%)	−10.0	−10.0	−5.0
$E_{t_0}[V_{t_0+\tau}]$ ($)	407 177	368 429	387 319
$\text{EaD}_{t_0+\tau}$ ($)	390 000	375 000	350 000
$E_{t_0}[\text{LGD}_{t_0+\tau}]$ (%)	21.7	26.3	17.0
$E_{t_0}[\text{LGD}_{t_0+\tau}] \times \text{EaD}_{t_0+\tau}$ ($)	84 617	98 678	59 511

(b) Conditional term structure of expected LGD and EaD incorporating expected prepayment

	Year 1	Year 2	Year 3
τ	1	2	3
$E_{t_0}[r^{\text{factor}}_{t_0,t_0+\tau}]$ (%)	−10.0	−10.0	−5.0
$E_{t_0}[V_{t_0+\tau}]$ ($)	407 177	368 429	387 319
$E_{t_0}[\text{LGD}_{t_0+\tau}]$ (%)	21.7	26.3	17.0
$\text{EaD}_{t_0+\tau}$ ($)	390 000	375 000	350 000
$E_{t_0}[\theta_{t_0+\tau}]$ (%)	7.0	10.0	14.0
$E_{t_0}[\text{EaD}_{t_0+\tau}]$ ($)	362 700	337 500	301 000
$E_{t_0}[\text{LGD}_{t_0+\tau}] \times E_{t_0}[\text{EaD}_{t_0+\tau}]$ ($)	78 706	88 763	51 170

to determine the expected EaD profile of our facility. The calculation will be quite straightforward if the EaD profile of the facility is deterministic over time. However, if the EaD profile is expected to be stochastic, we need to conduct further estimations for the credit conversion factors, as illustrated below.

5.1 Deterministic EaD

For lots of secured facilities (eg, residential mortgages, term loans, amortizing loans), EaD follows a deterministic path as dictated by the outstanding balance over the life of the loan. The EaD profile is known and prespecified at initiation (ignoring any prepayments or payments missed prior to potential default). There is, therefore, no uncertainty regarding the EaD value throughout the life of the facility, and thus $E_{t_0}[\text{EaD}_{t_0+\tau}]$ becomes $\text{EaD}_{t_0+\tau}$ when we evaluate (5.1) and (5.2).

Let us illustrate the one-year and lifetime EL estimation with a numerical example. Suppose the outstanding balance of a residential mortgage is currently $400 000. The remaining term is three years. According to the amortization schedule, the outstanding balances at the end of the next three years are \$390 000, \$375 000 and \$350 000, respectively. Today's appraised value (V_{t_0}) of the residential property is \$450 000. Suppose there is only a single underlying factor (eg, a national house price index) driving the value of the residential property. The estimated LGD parameters δ, α and β for this asset class are 0.75, 0.00 and 1.00, respectively. Suppose the annualized changes in the underlying house price index are expected to be -10%, -10% and -5% over the next year, the second next year and the third year, respectively. In Table 3(a), we report the predicted collateral values ($E_{t_0}[V_{t_0+\tau}]$) at the end of the next three years using (4.9) and based on the above expected changes in the underlying house price index. Then, using (4.11), we calculate the expected LGD profile ($E_{t_0}[\mathrm{LGD}_{t_0+\tau}]$) of this mortgage over the next three years, which is also presented in Table 3(a).

In the last row of Table 3(a), we report the expected loss (in dollars) of the mortgage contingent on the default occurring in each of the next three years. Further, suppose the one-year conditional PD of the mortgagor is 5% for each of the next three years. The one-year EL of this mortgage is therefore equal to $\$84\,617 \times 5\% = \$4231$, whereas its lifetime EL is

$$\$84\,617 \times 5\% + \$98\,678 \times 5\% \times 95\% + \$59\,511 \times 5\% \times 95\% \times 95\% = \$11\,604.$$

5.1.1 Incorporating prepayments

For certain products, the borrower is allowed to make prepayments up to a certain threshold.[19] In this case, we can use a more general case for $E_{t_0}[\mathrm{EaD}_{t_0+\tau}]$ when we evaluate (5.1) and (5.2), that is,

$$E_{t_0}[\mathrm{EaD}_{t_0+\tau}] = (1 - E_{t_0}[\theta_{t_0+\tau}]) \times \mathrm{EaD}_{t_0+\tau},$$

where $E_{t_0}[\theta_{t_0+\tau}]$ is the expected prepayment rate that can be estimated from historical data. In Table 3(b), we present the results of the previous numerical example while incorporating expected prepayment rate.

With the assumed prepayment rate, the one-year EL of the mortgage becomes equal to $\$78\,706 \times 5\% = \$3935$; whereas its lifetime EL is

$$\$78\,706 \times 5\% + \$88\,763 \times 5\% \times 95\% + \$51\,170 \times 5\% \times 95\% \times 95\% = \$10\,461.$$

5.2 Stochastic EaD

Many facilities have a stochastic EaD profile over time. For example, most of the time, the utilized amount (U) of a line of credit is only a fraction of its authorized amount (A). The decision to increase or decrease the drawn amount is largely at the discretion of the borrower. It is expected that the decision process is not completely

[19] Exceeding these preestablished thresholds may be subject to penalties.

random. For example, a generally bad business condition may induce more borrowers to draw down on their credit lines, whereas it should not be too surprising that the drawn amount tends to be lower in a booming market condition. More importantly, we expect borrowers to become more aggressive in utilizing their credit lines once default is imminent. We therefore expect the use of any undrawn amount (ie, the remaining balance of the authorized amount) to be highest during the year prior to the default event. In the Basel II A-IRB approach, in order to find out the expected EaD over the remaining life of the loan contract, it is typical to adopt a time series model based on the estimated "conversion rate" from the undrawn to the drawn amount over time, which is usually referred to as the credit conversion factor (CCF). Essentially, we assume facilities belonging to the same uniform segment (eg, a certain kind of line of credit) behave similarly in terms of how the authorized amount is utilized and thus share the same CCF. Specifically, if the undrawn amount at the end of last period is $(A_{t-1} - U_{t-1})$, the additional amount utilized (ΔU_t) in this period is assumed to be given by

$$\Delta U_t = (U_t - U_{t-1}) = \mathrm{CCF}_t \times (A_{t-1} - U_{t-1}), \tag{5.3}$$

ie,

$$U_t = U_{t-1} + \mathrm{CCF}_t \times (A_{t-1} - U_{t-1}). \tag{5.4}$$

Note that a subscript t is added to CCF to represent the most general case where CCF may vary over time according to the state of the business cycle. Note that the utilized amount at the end of period t (U_t) will become the EaD if the borrower defaults in period t. That is,

$$\mathrm{EaD}_t = U_{t-1} + \mathrm{CCF}_t \times (A_{t-1} - U_{t-1}). \tag{5.5}$$

In Basel II A-IRB, the risk horizon is one year, and therefore t is measured on a yearly basis. That is, EaD is defined over a one-year period.

As mentioned previously, we expect the conversion rate in the default year to be quite different from that in other years. We therefore use superscripts "D" and "ND" to differentiate the CCF in default year and nondefault year, respectively. Thus, we need to rewrite (5.4) and (5.5) as follows:

$$U_t = U_{t-1} + \mathrm{CCF}_t^{\mathrm{ND}} \times (A_{t-1} - U_{t-1}), \tag{5.6}$$

$$\mathrm{EaD}_t = U_{t-1} + \mathrm{CCF}_t^{\mathrm{D}} \times (A_{t-1} - U_{t-1}). \tag{5.7}$$

Equation (5.6) therefore represents the change in utilization amount during the years in which default has not occurred, whereas (5.7) presents how EaD is formulated in the default year. In general, we expect $\mathrm{CCF}_t^{\mathrm{D}}$ to be higher than $\mathrm{CCF}_t^{\mathrm{ND}}$.

The two CCFs can be estimated from historical data by tracking the variation of the utilization rates of the facilities over time. Facilities in each uniform segment (ie, a certain kind of facility) are assumed to share the same CCF. In A-IRB, CCF^{D} is typically assumed to be constant over time, ie,

$$\text{CCF}_1^{\text{D}} = \text{CCF}_2^{\text{D}} = \cdots = \text{CCF}_n^{\text{D}} = \text{CCF}^{\text{D}}. \tag{5.8}$$

This constant can be estimated from historical data by taking the average of the following ratios observed during the year prior to the respective default dates across all defaulted facilities of that uniform segment:

$$\frac{\text{EaD}_{\text{realized}} - U_{T_{\text{default}}-1}}{A_{T_{\text{default}}-1} - U_{T_{\text{default}}-1}}, \tag{5.9}$$

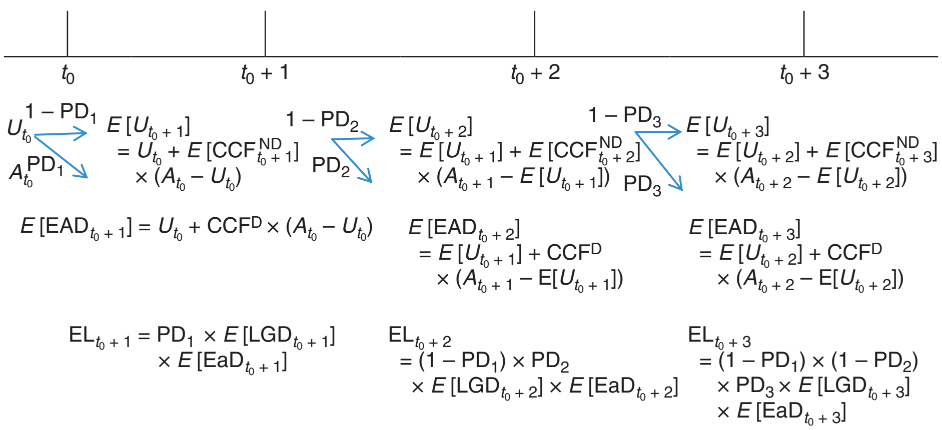
where $\text{EaD}_{\text{realized}}$ is the actual realized EaD of the defaulted facility (ie, the outstanding balance of the facility) at default time T_{default}, $U_{T_{\text{default}}-1}$ is the utilized dollar amount at $T_{\text{default}} - 1$ (ie, one year prior to the default date) and $A_{T_{\text{default}}-1}$ is the authorized dollar amount measured at the same time.

The estimation of the CCF for the nondefault year (CCF^{ND}) is likely to be more involved. We expect CCF^{ND} to vary with the business cycle. First of all, given more difficult business conditions, it is quite likely that borrowers will draw down more during bad years than in good years. On the other hand, it is also possible that, in order to take advantage of a good economy, borrowers in fact use more of their credit lines in good times than in bad times. In any case, the behavior is quite specific to the particular kind of facility. To satisfy IFRS 9, which calls for a conditional EaD assessment based on the CFCE, we need to model CCF^{ND} as a function of the prevailing business condition. Suppose we expect CCF^{ND} to be driven by the growth rate of real GDP. We can construct, say, three subsamples from our historical sample period based on the GDP growth rate realized in each year. In particular, the first subsample contains those years with a GDP growth rate within the fastest one-third of all years. In the second subsample, the GDP growth rate is within the middle third, and in the third subsample it is within the slowest third. We then calculate the average of the following ratios for all nondefaulting facilities within a uniform segment separately for each of the three subsample periods:

$$\frac{U_t - U_{t-1}}{A_{t-1} - U_{t-1}}, \tag{5.10}$$

where U_t and U_{t-1} are the utilized dollar amounts of the nondefaulting facility observed at time t and $t - 1$, respectively (ie, the two observations are exactly one year apart), and A_{t-1} is the authorized dollar amount at time $t - 1$. We therefore obtain three estimations of CCF^{ND} applicable to good, neutral and bad business conditions, respectively. In projecting the EaD profile for the EL calculation, we will therefore pick the appropriate CCF^{ND} based on our expectation of the prevailing real

FIGURE 3 Projected EaD term structure and EL over a three-year period.



GDP growth rate. Without the calibration of such a conditional $\mathrm{CCF}^{\mathrm{ND}}$, one conservative approach is to simply assume $\mathrm{CCF}^{\mathrm{ND}}$ takes the same value as the previously estimated $\mathrm{CCF}^{\mathrm{D}}$. This is conservative because $\mathrm{CCF}^{\mathrm{D}}$ is usually believed to be higher than $\mathrm{CCF}^{\mathrm{ND}}$. Using $\mathrm{CCF}^{\mathrm{D}}$ in projecting the changes in the utilized values for the nondefaulting years as well will therefore result in a larger EaD.

With the estimated CCFs, we can then project the expected utilized dollar amount and EaD for each facility within the same segment over the remaining life of the contract, based on the following recursive equations. Specifically, for the years prior to the default year, the expected utilized amount is given by

$$\begin{aligned} E[U_t] &= E[U_{t-1}] + E[\mathrm{CCF}^{\mathrm{ND}}_t] \times (A_{t-1} - E[U_{t-1}]) \\ &= E[U_{t-1}] \times (1 - E[\mathrm{CCF}^{\mathrm{ND}}_t]) + A_{t-1} \times E[\mathrm{CCF}^{\mathrm{ND}}_t], \end{aligned} \tag{5.11}$$

where we assume $E[\mathrm{CCF}^{\mathrm{ND}}_t]$ takes one of the three estimated values contingent on the expected real GDP growth rate at time t. If time t is the default year, then the expected EaD is given by

$$\begin{aligned} E[\mathrm{EaD}_t] &= E[U_{t-1}] + \mathrm{CCF}^{\mathrm{D}} \times (A_{t-1} - E[U_{t-1}]) \\ &= E[U_{t-1}] \times (1 - \mathrm{CCF}^{\mathrm{D}}) + A_{t-1} \times \mathrm{CCF}^{\mathrm{D}}, \end{aligned} \tag{5.12}$$

where we assume $\mathrm{CCF}^{\mathrm{D}}$ is constant over time. In applying (5.11) and (5.12), we further assume A is deterministic over time. Suppose today is time t_0 and the current utilized and authorized dollar amounts of the facility are U_{t_0} and A_{t_0}, respectively. Figure 3 illustrates the recursive calculations of the expected utilized amount and EaD term structure and the resulting EL over a three-year period.

TABLE 4 Expected utilized amounts and expected EaD over the next three years.

	Year 1	Year 2	Year 3
$E[\text{CCF}_t^{\text{ND}}]$ (%)	20	40	—
$E[U_t]$ ($)	60 000	76 000	—
$E[\text{EaD}_t]$ ($)	87 500	90 000	94 000

Let us end our discussion with a simple numerical example. Suppose there is a line of credit for which we would like to calculate the EL over the next three years. The current utilized amount is \$50 000. Suppose the authorized amount is fixed at \$100 000 for the remaining life of the contract. The estimated CCF^{D} for the segment is 75%. There are three levels of CCF^{ND} estimated contingent on three different states of the economy: $\text{CCF}^{\text{ND}} = 10\%$ (high growth rate of real GDP), $\text{CCF}^{\text{ND}} = 20\%$ (moderate growth rate of real GDP) and $\text{CCF}^{\text{ND}} = 40\%$ (low growth rate of real GDP). Suppose that, consistent with CFCE, the economic outlook is such that the GDP growth rate is expected to be moderate in the upcoming year and then become slower in the subsequent years. Using (5.11) and (5.12), we calculate the expected utilized amounts and expected EaD for this line of credit for the next three years and report the results in Table 4.

Further, suppose the one-year conditional PD of the borrower is 5% for each of the next three years, and the expected LGD is always 50%. The three-year EL of this line of credit is therefore

$$\begin{aligned}\$87\,500 \times 50\% \times 5\% + \$90\,000 \times 50\% \times 5\% \times 95\% \\ + \$94\,000 \times 50\% \times 5\% \times 95\% \times 95\% = \$6446.\end{aligned}$$

5.3 Products with noncontractual maturities

Term to maturity is also an important variable in IFRS 9, considering that, in bucket 2, EL is estimated over the "life-time" of the loan. Our discussion in this paper is limited to loans with contractual maturities. There is also an extensive debate on how to deal with products with noncontractual maturities. An example would be credit cards. A-IRB calls for the use of a one-year term to maturity for retail products. This could be a result of banks' argument that they maintain the option to cancel the credit card at any point in time if they are not satisfied with the creditworthiness of the cardholder. An "effective" term to maturity can be estimated from historical data for homogeneous segments of a given product. We can estimate the average card holding period, for example, for credit cardholders with a credit score over a certain threshold, which may be different (longer) than those with a lower credit score. An interesting problem arises: when a bank estimates EL conditional on an expected macroeconomic outlook, not only PDs and LGDs (and EaD in certain cases), but also the effective term to maturity (thus the lifetime over which the EL will be calculated) will change with respect to the outlook.

Another product to consider is the revolvers, where there is an immediate maturity, upon arrival of which the loan can be extended (or renewed), provided that the conditions defined at origination are met. The question is whether the immediate maturity (the next renewal date) or the final maturity date (the maturity date if the loan is extended all the way) should be used as the effective maturity date. The implications are, of course, very significant, given that the next renewal date can be less than a year, whereas the final maturity date can be more than five or seven years. This is a topic currently being discussed by the BCBS. It is already accepted by the BCBS that if the loan is "unconditionally cancelable" by the bank at the next renewal date, the effective term to maturity is simply the next renewal date. The banks argue that this treatment be extended to loans that are "conditionally" cancelable by the bank. They argue that, should the borrower not meet the predefined performance targets (eg, meeting covenants, reaching a certain stage of development with the construction loans), the bank has the right to cancel the loan, and thus the next renewal date should be used as the effective maturity date rather than the final maturity date. The BCBS's decision on the matter is expected over the next few months and the decision can be considered in the IFRS 9 debate as well. The banks have always had the option to estimate the effective maturity from historical data for homogeneous segments. This would likely produce an effective maturity date in between the next renewal date and the final maturity date.

The above argument would also hold for credit cards, as we expect that effective maturity will be conditional on the expected economic outlook for the revolvers as well. Certain behavioral characteristics can be observed from historical data for certain products and borrowers. For example, during the oil market downturn in Alberta (Canada), borrowers in the oil and gas servicing sector proactively sold their tangible assets and paid off their debt, and thus shortened their effective term to maturity. As a counterexample, obligors in other industries without such tangible assets can indeed try to extend the duration of the loan in order to survive. Some banks more readily exercise the conditional cancelations clause (and thus shorten the effective maturity) than others who find that the cancelation of the loan forces a default and actually increases the losses. This topic certainly warrants future study.

6 CONCLUSIONS

In this paper, we examine how we may utilize the A-IRB PD, LGD and EaD models for IFRS 9 and show how we can arrive at the expected one-year or lifetime credit loss with the PD, LGD and EaD measures obtained from these models. We highlight the necessary model adaptations required to satisfy the new accounting standard of impairment measurement. By leveraging the A-IRB models, banks can lessen their modeling efforts in fulfilling IFRS 9 and capture the synergy among different modeling endeavors within the institutions. In introducing the proposed PD, LGD and EaD models, we provide detailed examples of how they may be implemented on secured lending.

To circumvent the need to assign probabilities to future scenarios in evaluating the expected PD in scenario analysis, we propose a convexity adjustment approach to deal with the nonlinear relation between conditional PD and the underlying macroeconomic drivers. By doing so, we can enhance the objectivity and replicability of the resultant EL measure.

DECLARATION OF INTEREST

The authors report no conflicts of interest. The authors alone are responsible for the content and writing of the paper. The opinions expressed in this paper are those of the authors and are not necessarily endorsed by the authors' employers.

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