Default Risk Charge: Modeling Framework for the “Basel” Risk Measure

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Appendix

A. Correlation Model: Data

Table A.1 below provides a breakdown of the 1,145 equities used in the calibration of the equity correlation model. The index constituents of Eurostoxx 50, SMI, FTSE-100, S&P500, ASX-200, HSI and Nikkei-225 reflect the index compositions as of end of December 2013. Source: *Bloomberg* (the country reflects the 'country of risk', the industry the 'industry sector'). The sample consists of about 40% of U.S.-based and Asian-Pacific firms each, while European names make up about 15%. The industry split is reasonably balanced across the different categories.

Table A.1: Equity data for correlation modeling

	Euro-stoxx 50	SMI	FTSE-100*	S&P 500	ASX-200**	HSI	Nikkei-225	All
Total	50	20	101	500	199	50	225	**1145**
Country								
Australia	-	-	-	-	186	-	-	**186**
Belgium	1	-	-	-	-	-	-	**1**
China	-	-	-	-	-	27	-	**27**
France	19	-	-	-	-	-	-	**19**
Germany	14	-	-	-	-	-	-	**14**
Hong Kong	-	-	-	-	-	22	-	**22**
Ireland	1	-	2	1	1	-	-	**5**
Italy	5	-	-	-	-	-	-	**5**
Japan	-	-	-	-	-	-	225	**225**
Jersey	-	-	1	-	-	-	-	**1**
Mexico	-	-	1	-	-	-	-	**1**
Netherlands	4	-	2	-	-	-	-	**6**
New Zealand	-	-	-	-	6	-	-	**6**
Singapore	-	-	-	-	1	-	-	**1**
Spain	6	-	-	-	-	-	-	**6**
Switzerland	-	19	2	1	-	-	-	**22**
United Kingdom	-	-	92	2	1	1	-	**96**
United States	-	1	1	496	4	-	-	**502**
Industry								
Basic Materials	2	2	9	25	34	1	32	**105**
Communications	4	1	7	39	21	3	11	**86**
Consumer Cyclical	5	2	17	69	23	6	44	**166**
Consumer Non-cyclical	8	6	24	100	30	3	30	**201**
Diversified	-	-	-	1	1	4	-	**6**
Energy	3	1	7	45	14	6	3	**79**
Financial	13	5	20	80	40	21	26	**205**
Industrial	8	3	10	65	28	1	66	**181**
Technology	2	-	2	46	1	1	8	**60**
Utilities	5	-	5	30	7	4	5	**56**

* The FTSE-100 consists of 100 companies, but there are 101 listings since Royal Dutch Shell has both A and B class shares listed.

** With only quarterly rebalancing, the number of names in the index can slightly diverge from 200.

B. Correlation Model: Calibration Details

Country and industry factors

Table B.1 shows the results of the regression analysis run on the basis of equation (3.1) in the main article for the identified stress period, providing evidence that most of the country and industry returns are moving in line with the global returns.

Table B.1:
Equity return factor model – Country and industry factors

This table provides the results of the country and industry factor analysis derived from the equity returns of the index constituents of Eurostoxx 50, SMI, FTSE-100, S&P500, ASX-200, HSI and Nikkei-225. The equity index compositions reflect the status as of end of December 2013. The analysis is based on non-overlapping monthly (log-)returns and conducted over the period September 2007 through September 2010 that has been identified as the one with the most "stressed" correlation (see Figure 1; 35 return periods). After standardization the equity returns are used to derive global, country and industry factors as cross-sectional averages at each time point. The country and industry factors are regressed onto the global factor; coefficients (β_C and β_I, respectively) as well as R^2 are provided in the table, complemented by the standard deviation of the residual returns (see also equation (3.2) in the main article). t-tests are conducted to evaluate whether the coefficients differ from one; *** (**, *) indicates significance [two-sided test] at the 1% (5%, 10%) level.

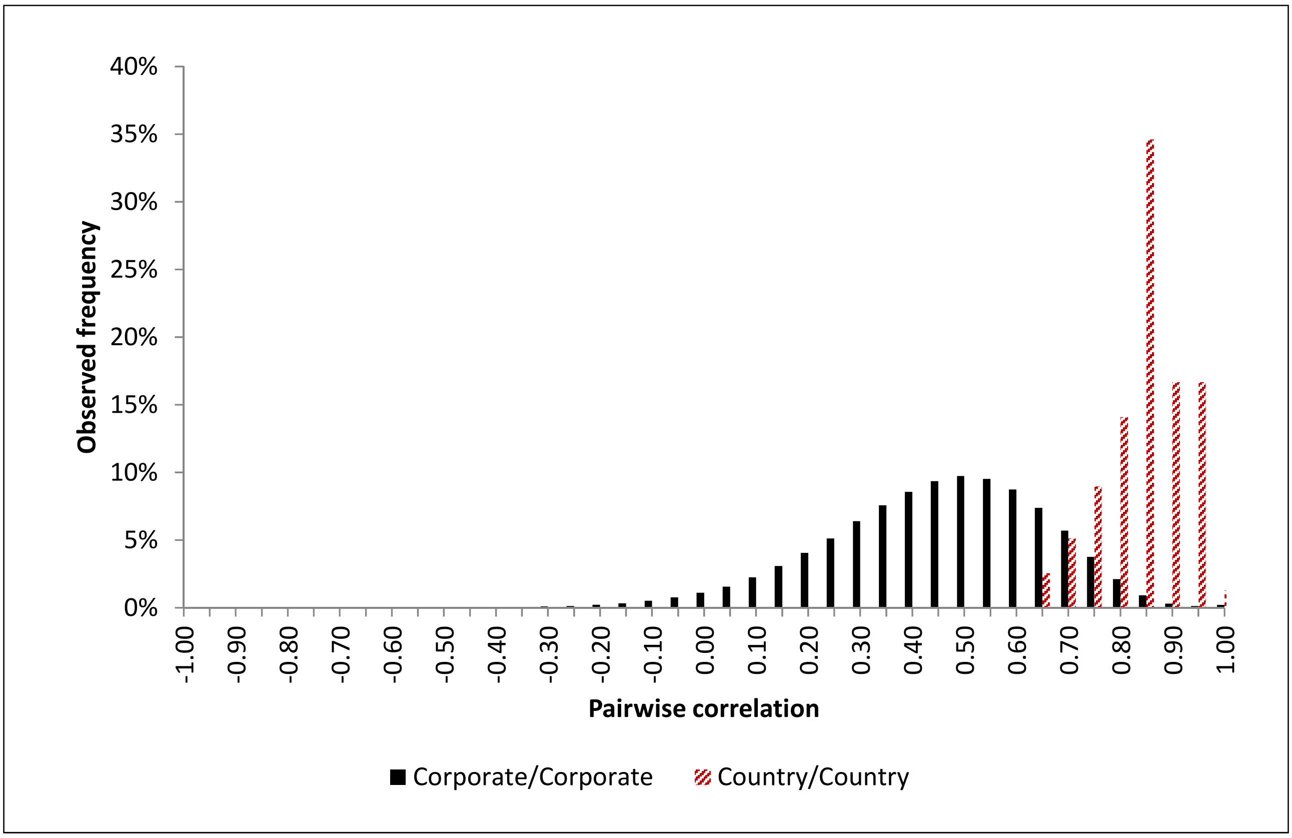
Country	β_C	t-value	R^2	σ_C	σ_G
Australia	0.8573	-2.268**	84.5%	24.2%	
Belgium	-	-	-	-	
Switzerland	1.0267	0.466	90.9%	14.0%	
China	0.9945	-0.054	74.2%	15.2%	
Germany	1.0968	1.580	90.4%	18.7%	
Spain	1.1881	1.768*	78.6%	32.8%	
France	1.0384	0.480	83.2%	37.5%	
United Kingdom	0.9485	-0.764	85.3%	20.9%	
Hong Kong	1.0380	0.419	79.4%	10.6%	65.9%
Ireland	0.8838	-1.065	67.2%	20.3%	
Italy	1.2470	2.597**	83.5%	40.3%	
Jersey	-	-	-	-	
Japan	1.0399	0.437	81.2%	31.1%	
Mexico	-	-	-	-	
Netherlands	0.9970	-0.027	70.4%	42.6%	
New Zealand	-	-	-	-	
Singapore	-	-	-	-	
United States	1.0424	1.120	95.7%	14.6%	

Industry	β_I	t-value	R^2	σ_I	σ_G
Basic Materials	1.0111	0.188	89.7%	22.6%	
Communications	0.9796	-0.562	95.5%	14.0%	
Consumer Cyclical	1.0370	0.934	95.3%	15.2%	
Consumer Non-cyclical	0.8668	-2.739***	90.3%	18.7%	
Diversified	1.1938	2.268**	85.2%	32.8%	65.9%
Energy	0.9620	-0.390	74.1%	37.5%	
Financial	1.0540	0.995	91.7%	20.9%	
Industrial	1.0965	3.492***	97.9%	10.6%	
Technology	1.0794	1.506	92.5%	20.3%	
Utilities	0.8345	-1.578	65.1%	40.3%	

Empirical correlation structure and model fit

The empirical distribution that the factor model is supposed to reproduce is illustrated in **Figure B.2**. It shows the distribution of the corporate/corporate equity correlations during the identified stress period. Additionally, based on the country returns defined as cross-sectional averages through time, the distribution of country/country correlations is displayed.[1] The comparatively high country/country correlations (on average around 82%) point towards a dominant common (global) driver of the returns. These results are broadly consistent with the findings in Longstaff *et al.* (2011) who analyze the main determinants of sovereign CDS spreads and their correlation structure. Over the crisis period of 2007-2010 and using monthly returns, they report an average sovereign CDS spread correlation of 73%. They also find a significant global factor (first factor in a principal component analysis) that explains about 75% of the sovereign CDS spread returns.

1 For a qualifying country return time series, at least five valid returns (out of 35) are required.



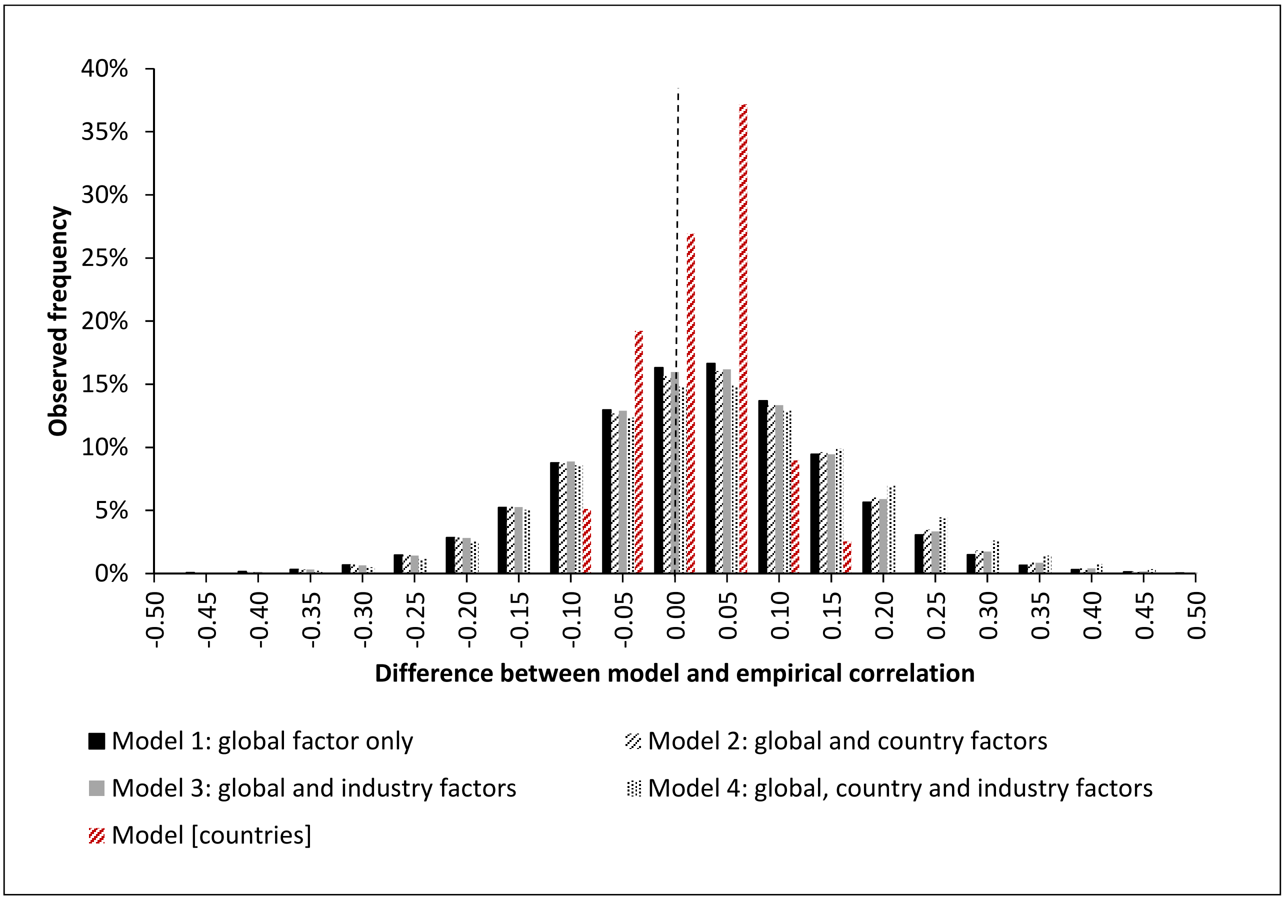
The figure shows the distribution of the equity correlation during the identified stress period (September 2007 through September 2010), based on the index constituents of Eurostoxx 50, SMI, FTSE-100, S&P500, ASX-200, HSI and Nikkei-225. The equity index compositions reflect the status as of end of December 2013. The return correlation is calculated from non-overlapping monthly (log-)returns. Besides the corporate/corporate correlations, country/country correlations are derived from the corresponding cross-sectional averages of corporate returns.

Figure B.2: Distribution of correlations during the stress period

The final judgment as to whether the proposed model is suitable is provided by the differences between model-implied and empirically measured (pairwise) correlations.[2] Model-implied corporate/corporate correlations are calculated on the basis of equation (3.3) in the main article and the corresponding estimated parameters, assuming zero correlations between residuals of different obligors. Similarly, model-implied country/country correlations are calculated on the basis of the country

2 It is worth pointing out that *"[a] bank must validate that its modelling approach for these correlations is appropriate for its portfolio, including the choice and weights of its systematic risk factors"* (Basel Committee on Banking Supervision (2016), p. 61).

returns equation in (3.1) in the main article and the corresponding estimated parameters, assuming zero correlations between residuals of different countries. The subsequent **Figure B.3** below shows the resulting distributions, differentiated by corporate/corporate and the associated four model variants and the country/country case.[3] To a varying degree, on average, the suggested models reflect well the empirical corporate/corporate and country/country correlations.



The figure shows the distribution of the differences between the model-implied and empirical (pairwise) correlations. As for the model, the four variants (global factor only, global/country, global/industry, global/country/industry factors) are explored for the corporate/corporate correlations. The differences between the model-implied and empirical country/country correlations are illustrated by means of a separate histogram.

Figure B.3: Distribution of differences between model and empirical correlations

3 Recall that all correlation pairs are equally weighted in this analysis.

C. Correlated Defaults: Simulation

The estimated R^2 parameters and the coefficients from regressions in (1) and (2) in the main article can be used to simulate correlated returns of different sovereigns and corporate obligors and subsequently correlated defaults. For a sovereign obligor, the simulated return is chosen as the corresponding country return. Specifically, in order to simulate the – standardized – country returns $\breve{r}_{C(j)}$, one would simulate a global factor Z_G and a country-specific factor $Z_{C(j)}$, both assumed to be independent and standard normally distributed, and rescale these using the estimated country R^2 parameters:

$$\breve{r}_{C(j)} = \text{sgn}(\hat{\beta}_{C(j)})\sqrt{R^2_{C(j)}}Z_G + \sqrt{1 - R^2_{C(j)}}Z_{C(j)} \tag{i}$$

with $\text{sgn}(\text{x})$ as the sign function. $\hat{\beta}_{C(j)}$ and $R^2_{C(j)}$ are the estimates from **Table B.1**. The following straightforward expression can be used to calculate the country/country correlation implied by the estimated model in (1) in the main article:

$$\text{Corr}\left(\breve{r}_{C(j_1)}, \breve{r}_{C(j_2)}\right) = \left(1 - R^2_{C(j_1)}\right)\mathbf{1}_{\{j_1=j_2\}} + \text{sgn}(\hat{\beta}_{C(j_1)}\hat{\beta}_{C(j_2)})\sqrt{R^2_{C(j_1)}R^2_{C(j_2)}} \tag{ii}$$

Similarly, in order to simulate the – standardized – return $\breve{r}_i$ of a corporate from country $C(i)$ and industry $I(i)$, one would simulate a global factor Z_G and the relevant country-specific, industry-specific and idiosyncratic factors, $Z_{C(i)}$, $Z_{I(i)}$ and ε_i, all assumed to be independent and following standard normal distributions:[4]

$$\breve{r}_i = \text{sgn}(\hat{\beta}_i)\sqrt{\frac{R^2_i}{\hat{\Psi}_i}}\left(\hat{\gamma}_G\hat{\sigma}_G Z_G + \hat{\gamma}_{C(i)}\hat{\sigma}_{C(i)}Z_{C(i)} + \hat{\gamma}_{I(i)}\hat{\sigma}_{I(i)}Z_{I(i)}\right) + \sqrt{1 - R^2_i}\varepsilon_i \tag{iii}$$

4 Note that this expression assumes that all country and industry residual factors are independent from each other, which is a simplification, albeit backed up empirically: the average correlation among the residual country (industry) factors in the setup presented here amounts to about 2% (-8%), the average correlation between residual country and industry factors to approximately -1%.

where $\hat{\beta}_i$ and R_i^2 represent the coefficient estimates based on equation (3.3) in the main article; $\hat{\gamma}_G, \hat{\sigma}_G, \hat{\gamma}_{C(i)}, \hat{\sigma}_{C(i)}, \hat{\gamma}_{I(i)}, \hat{\sigma}_{I(i)}$ are the estimates based on equations (3.1) and (3.2); $\hat{\Psi}_i = \hat{\gamma}_G^2 \hat{\sigma}_G^2 + \hat{\gamma}_{C(i)}^2 \hat{\sigma}_{C(i)}^2 + \hat{\gamma}_{I(i)}^2 \hat{\sigma}_{I(i)}^2$ is a normalization coefficient.

Based on (iii), the pairwise correlation between any two corporate returns can be expressed in closed form as:

$$\operatorname{Corr}(\breve{r}_{i_1}, \breve{r}_{i_2}) = \left(1 - R_{i_1}^2\right) \mathbf{1}_{\{i_1 = i_2\}} + \operatorname{sgn}\left(\hat{\beta}_{i_1} \hat{\beta}_{i_2}\right) \sqrt{\frac{R_{i_1}^2 R_{i_2}^2}{\hat{\Psi}_{i_1} \hat{\Psi}_{i_2}}} \cdot \left(\hat{\gamma}_G^2 \hat{\sigma}_G^2 + \hat{\gamma}_{C(i_1)}^2 \hat{\sigma}_{C(i_1)}^2 \mathbf{1}_{\{C(i_1) = C(i_2)\}} + \hat{\gamma}_{I(i_1)}^2 \hat{\sigma}_{I(i_1)}^2 \mathbf{1}_{\{I(i_1) = I(i_2)\}}\right). \tag{iv}$$

The integration with the marginal default probabilities is straightforward. Specifically, the simulated standardized asset return $\breve{r}_i$ is used to define an overall asset return V_i as follows:

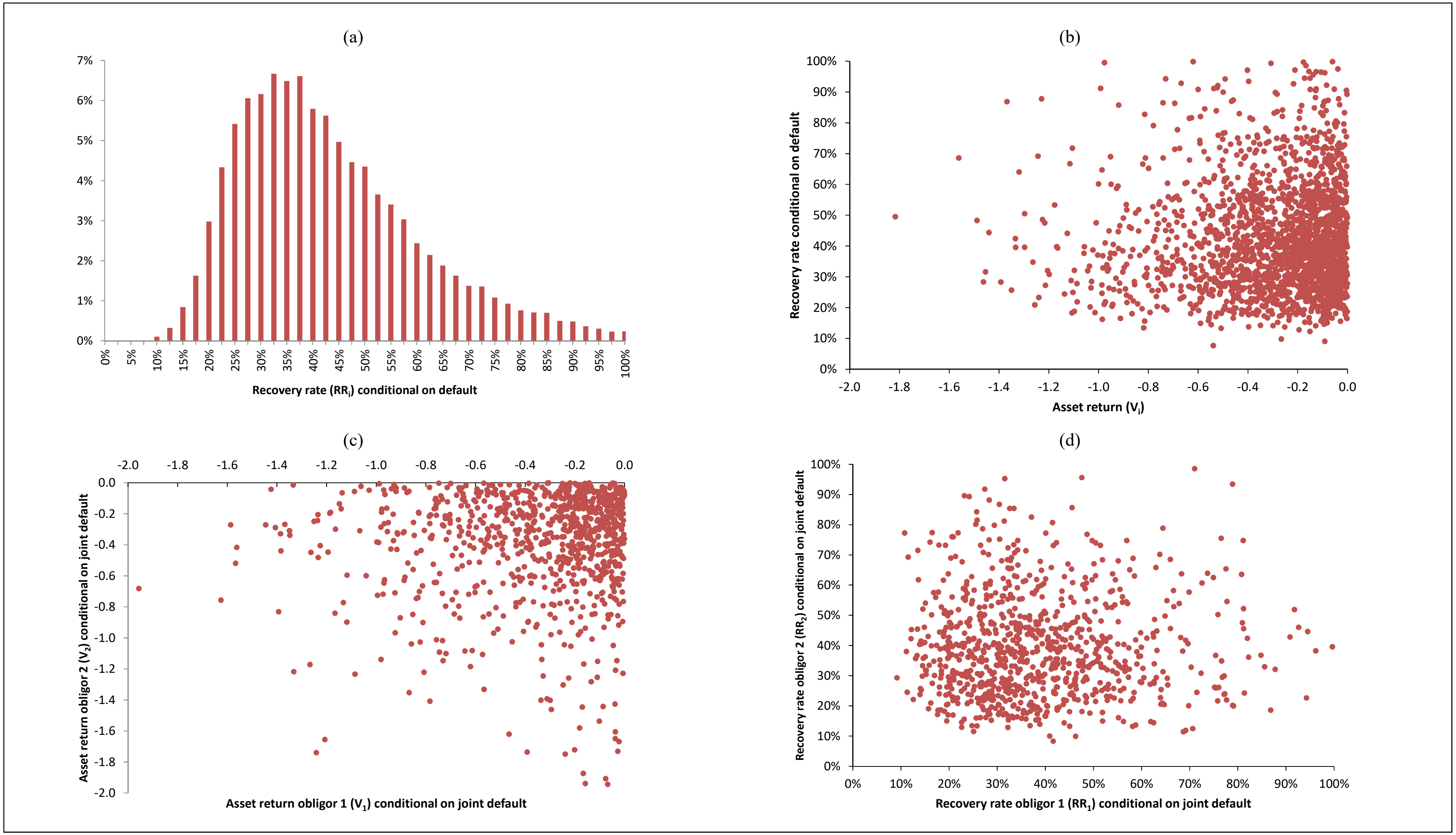
$$V_i = \Lambda' x_i + \breve{r}_i \tag{v}$$

where the deterministic vector $\Lambda = (\Lambda_1^{\text{Corp}}, \Lambda_2^{\text{Corp}}, \ldots, \Lambda_L^{\text{Corp}}, \Lambda_1^{\text{Sov}}, \Lambda_2^{\text{Sov}}, \ldots, \Lambda_L^{\text{Sov}})$ codifies the default probabilities PD_l for corporates and sovereigns across the spectrum of L ratings according to **Table 1 in the main article.** This is achieved by setting $\Lambda_l = -N^{-1}(\text{PD}_l)$ with N^{-1} as the inverse of the cumulative standard normal distribution function. The $x_i = (\ldots, 0, 1, 0, \ldots)$ indicate the type (corporate or sovereign) and rating of obligor i. In this setup, the overall asset return V_i has a default threshold equal to zero, i.e., a default is triggered when $V_i < 0$.

D. Detailed Analysis of the Asset-Recovery Model

In order to study the asset-recovery rate model more closely, **Figure D.1** provides an insight into the properties for the one- and two-obligor case. Panel (a) shows the marginal distribution of recovery rates that the model produces conditional on default. This largely preserves a log-normal distributional shape. Panel (b) illustrates the asset-recovery rate dependence conditional on default. In line with the model philosophy a more negative asset return tends to be associated with a lower recovery rate. The strength of the correlation depends on the assumed level of ρ^{Y}. Given the assumed level of ρ^{Y} of about 4%, the unconditional correlation between the asset return and recovery rate is about 13% while conditional on default it amounts to only about 5%. In order to analyze the model behavior across obligors Panel (c) shows the asset correlation between two names, conditional on a joint default. The dependence is less pronounced than the unconditional parameterization ($\rho^{V,\mathrm{Corp}}$ of about 43%) suggests. Panel (d) provides an illustration of the joint recovery rates conditional on two obligors defaulting. In line with the assumed low value for ρ^{Y}, the correlation between recovery rates conditional on joint defaults is close to zero.

For sovereigns, similar qualitative effects with respect to recovery-default correlation as shown in **Figure D.1** for corporates are observed. The correlation between the asset return and (log-)recovery rates for sovereign obligors ($\rho^{VY,\mathrm{Sov}}$ around 18%) is comparable to the corporate case ($\rho^{VY,\mathrm{Corp}}$ around 13%).



The figure provides an illustration of the calibrated asset-recovery rate model on the example of one and two BBB-rated corporate obligors. Panel (a) shows the marginal distribution of recovery rates that the model produces conditional on default. Panel (b) illustrates the asset-recovery rate dependence conditional on default. In Panel (c) the asset correlation between two obligors is shown, conditional on a joint default. Panel (d) provides an illustration of the joint recovery rates conditional on both obligors defaulting.

Figure D.1: Analysis of asset-recovery rate model

Concerning the modeling and estimation of the recovery rate and the default probabilities for sovereign debt, one would likely want to differentiate between local currency and foreign currency issued debt since, from a foreign investor's point of view, the former carries both default and foreign exchange rate risk (not to be reflected in the DRC) due to the expected devaluation of the local currency in the event of default. While, for example, Du and Schreger (2013) examine the properties and driving factors in a comprehensive way, very limited historical data for defaults and recovery rates on local currency debt (confer Moody's (2011)) renders largely qualitative or relative assessments in the calibration of recovery rates for local versus foreign currency debt necessary. One possibility would be to use the different ratings for local currency and foreign currency debt issued by sovereign entities (as provided by Standard & Poor's) to derive expected recovery rates for such debt.

E. P&L Generation: Influence of Default Times

While the DRC model is to assume constant positions over the one-year horizon, to deny any benefit from recognizing a dynamic hedging strategy, it is also supposed to *"capture any material mismatch between a position and its hedge"* (Basel Committee on Banking Supervision (2016), p. 62). The latter is supposed to reflect, besides any potential basis effects, the risk arising for long/short positions from the *timing of defaults* within the one-year capital horizon. So if a long position in a one-year bond were to be hedged with a credit default swap on the issuer expiring in three months, the DRC is expected to take into account scenarios where a loss could occur from the obligor's default between months four and twelve. Additionally, in the case of equity positions, the banks have the *"discretion of applying a minimum liquidity horizon of 60 days to the determination of default risk charges for equity sub-portfolios"*. As the constant position assumption is to be maintained, it means that for such designated equity sub-portfolios, the default risk capital charge should capture losses over a 60-days horizon. These losses would then be aggregated with those over longer horizons applied to other sub-portfolios in the overall capital calculation.

With the probable intention to simplify the setup compared to the IRC the maturity dimension translates into assuming a constant position over $\min\{\text{maturity, one year}\}$ for each position in the portfolio and thus renders the introduction of a multi-step simulation model necessary. After a usually equidistant partitioning of the one-year horizon following an agreed granularity (say, one or three months) the simulation model needs to be amended to reflect those time-steps. There are a range of ways and empirical evidence on the probability of default for sub-periods and results can vary by a large degree (see, for example, the discussion in Smillie and Epperlein (2007)). One straightforward way is to use a geometric scaling to derive the default probability for each time period $i = 1, 2, \ldots, n$ as $\mathrm{PD}_i = 1 - (1 - \mathrm{PD}_{\mathrm{ann}})^{1/n}$ from the annual value $\mathrm{PD}_{\mathrm{ann}}$. Assuming an annual default probability of 5% the above case of a bond hedged with a CDS would lead to a zero P&L in a one-step time model, but a loss of (one minus recovery rate) in about 3.7% of the cases in a model with at least quarterly time-step granularity. The multi-step model also allows for a consistent default risk simulation for obligors with equity and credit instruments attracting different liquidity horizons. The default probability for the credit instrument over the first 60 days should be the same as for the equity instrument given the same underlying obligor.

A side effect of the multi-step setup is the well-known fact that the effective correlation between obligors will reduce as the number of time steps increases (see Dunn (2008), among many others) – widely referred to as “correlation wash-out”. Assume, for example, a long/short position in bonds of two obligors with the same (non-stochastic) recovery rate, a probability of default of 10% p.a. each and an (equity) correlation of 45%. The probability of a zero P&L at a one-year horizon is about 86% for a yearly time-step granularity, but reduced to about 83% for a monthly granularity; the remaining probability of 14% (17%) reflects the cases of only one of the obligors defaulting over the one-year period. As a tendency, the higher the probability of default the more pronounced the effect. There is usually no easy means of overcoming the influence of the discretization on the correlation. The latter is a large matrix in practice, defined by the outlined factor notation. If one were to target the preservation of the correlation from the one-step setup one could, for instance, numerically determine an equivalent *n*-step correlation that replicates – for a given population of obligors with usually different probabilities of default and pairwise correlations – the average. One simplified way of achieving this consists in leaving the factor structure unchanged and transforming the R^2 values from the Section C, for example, by means of a logit or arctan function in order to preserve the natural boundaries.

F. Example Portfolios: Analysis of Results from Standardised Approach

To explore the results in more detail, it is worth noting that the SA for the default risk is defined as an aggregation of the individual jump-to-default (JTD) amounts in a portfolio weighted by rating-based default risk contributions.[5] For long-short portfolios the SA allows for netting between the long and short positions, with this benefit depending on the relative weight of the long positions' JTD to the total absolute aggregated JTD. The losses given default (and thereby recovery rates) used in the calculation of the JTD are also prescribed in the SA.[6] The SA-based DRC can be seen as an expected loss measure for a portfolio with stressed default probabilities, which are the prescribed default risk weights, and prescribed losses given default. This measure therefore does not depend explicitly on the correlation between obligors in the portfolio, but is linearly dependent on the number of obligors in the portfolio and the ratio of the aggregate long JTD to the total aggregated JTD. The latter is a rough measure of netting benefits and it does not capture the true default correlations in the underlying portfolio. Unlike the SA, the DRC based on a portfolio model corresponds to an extreme tail measure of the simulated P&L distribution and thereby depends on the default correlations in the portfolio and the size of the portfolio, in addition to the assumed marginal default probabilities and recovery rates.

To illustrate the impact of different factors on the two methods – model-based versus SA-based – for calculating the DRC, consider the example of a hypothetical equity portfolio with all obligors rated BBB. The recovery rate is 0% in both approaches. Assume first a long-only equity portfolio for which the results are illustrated in Panel (a) of **Figure F.1** below. For a zero correlation between returns (defaults), the model-based DRC is smaller than the SA-based DRC regardless of the portfolio size.

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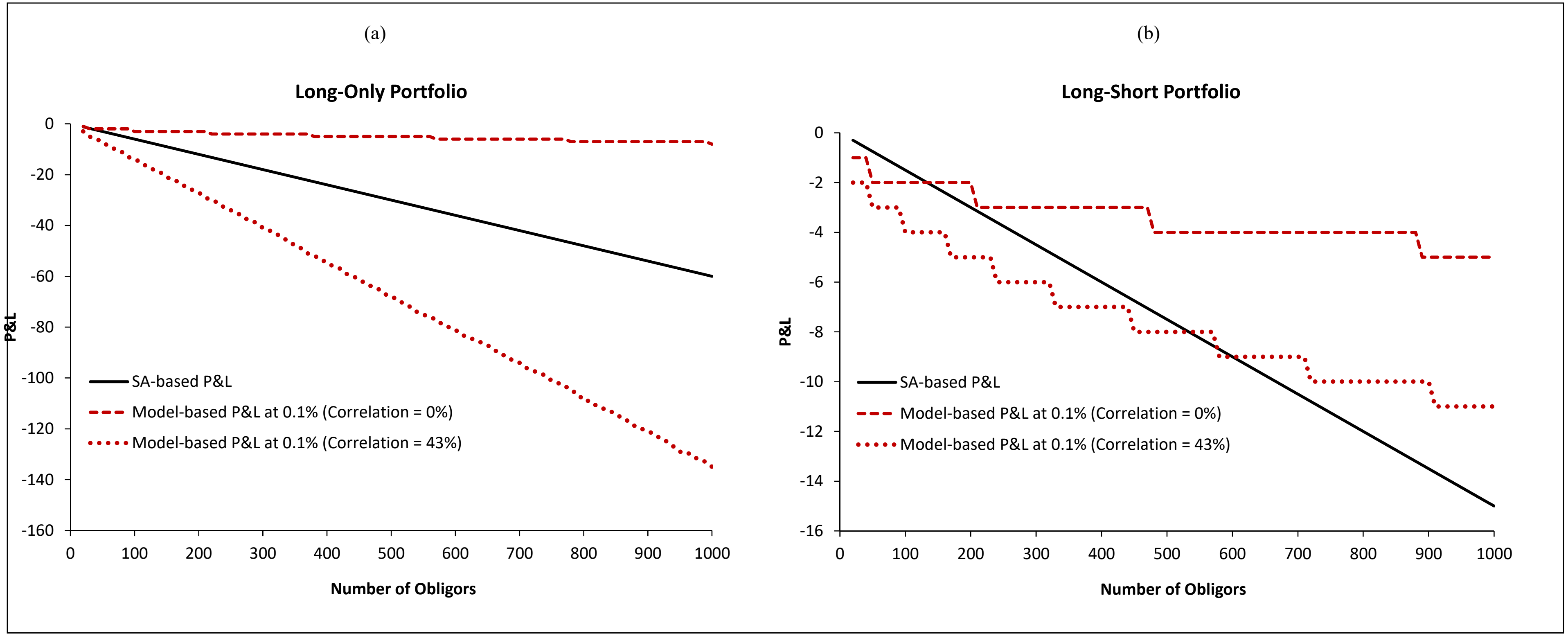
5 The default risk weights vary from 0.5% for AAA-rated names to 50% for CCC-rated and 100% for defaulted names.

6 According to the ruleset, the LGD under the SA is 100% for equity instruments and non-senior instruments, 75% for senior debt instruments and 25% for covered bonds (see Basel Committee on Banking Supervision (2016), p. 43).

The difference between the two measures increases with the number of securities in the portfolio. This is due to the significant diversification benefit of adding uncorrelated securities in a portfolio model when compared to the SA that just sums the weighted JTD for all names in the portfolio. For a return correlation of 43% – corresponding to the one-factor ("global factor only") model in this article –, the opposite holds: the model-based DRC is always larger than the SA-based one. This is because the portfolio diversification benefits of adding highly correlated securities to a long-only portfolio are reduced comparatively.

Consider next the example of a portfolio with all obligors being rated BBB, but a 50% ratio between the long JTD to total JTD. The results for the long-short portfolio are illustrated in Panel (b) of **Figure F.1**. The model-based DRC is a step-like function of the portfolio size and tends to be larger than the SA-based DRC for small portfolios but smaller than the SA-based DRC for large portfolios. The size of the portfolio for which the two DRC measures are equal depends on the correlation assumption. For a zero correlation between returns, the model-based DRC is larger than the SA-based one for a BBB-only portfolio with less than around 150 obligors, and smaller otherwise. For a default correlation of 43%, the portfolio size above which the SA-based DRC is larger is around 550 obligors. This explains the findings for the example portfolios in **Table 4 in the main article,** where the SA-based DRC is always smaller than the model-based DRC given that the maximum portfolio size is 259 securities (*portfolio J*).



The figure shows the comparison between the model- and SA-based DRC for an equity portfolio of BBB-rated obligors as a function of the number of obligors in the portfolio and the correlation assumption in the model-based DRC. A unit nominal (=1) and a recovery rate of 0% are assumed for each position. Panel (a) illustrates the results for a long-only portfolio. Panel (b) provides the results for a long-short portfolio with a 50% ratio between the long absolute JTD to the total absolute JTD.

Figure F.1: Comparison between model-based and standardised approach for DRC

G. Example Portfolios: Sensitivity Analysis

Table G.1 below shows selected sensitivity analyses of the DRC figures with respect to changes in (a) obligor credit quality, (b) average recovery rates and (c) average default correlation.

In particular, in case (a) the obligor ratings are each decreased and increased by one notch, to attract a different default probability according to **Table 1 in the main article**.[7] For Portfolios A, C, E and H, as expected, the DRC is monotonically increasing (decreasing) with the rating quality worsening (improving). The DRC increases to 35% for these portfolios if all the obligors are downgraded by one notch. In the opposite case where all obligors are upgraded by one notch, the decrease in DRC varies between 15% and 23%, hence generally smaller in absolute terms than in the case of rating downgrades. For the long/short Portfolios B, D, F and I, the influence of changes to the initial ratings is qualitatively similar although theoretically less obvious. For the equity Portfolio I the DRC does not change for a one-notch downgrade or upgrade. For Portfolios B, D, F the effects are similar to the long portfolios.

When stressing the (average) recovery rates, case (b), the picture is as follows. The recovery rate plays a role only in case of a default; therefore, there is an obvious link between the effects from recovery rates and ratings (default probabilities). A relative change in the average recovery rate of 25% has a significant effect on DRC. The change in DRC due to an increase or decrease in average recovery rate is quite symmetric and homogeneous across different portfolios. This can be explained by analogy to the case of constant recovery rates for which a relative change scenario would result in a similar change in the portfolio risk measure (homogeneity property).

The default correlation assumption and its sensitivity to changes are analyzed as case (c). Two changes in correlation are considered: one where the average correlation is decreased by 10% and

7 The worst attainable rating equals CCC/C, reflecting the last entry in Table 1.

another one where the average correlation is decreased by 25%.[8] This case also addresses the question with regard to the behavior of DRC in periods of stress. In all long-only portfolios – Portfolios A, C, E and H – the DRC is monotonically increasing in correlation, i.e., the higher the correlation the less diversification benefit the portfolio offers through its constituents. Moving on to the long/short Portfolios B, D, F, G, I and J, the picture is qualitatively similar, however, the magnitude of the change in DRC is significantly reduced compared to the corresponding long-only portfolios. For example, for the high-yield corporate Portfolio C, the two changes in correlation are associated with decreases in DRC of around 8% and 19%, respectively, compared to 3% and 6% for the long-short high-yield portfolio D. For Portfolios F and I, a decrease in correlation has (almost) no effect on the DRC.

Notably, in general, the correlation influence on the DRC is not that obvious to predict. Starting with a theoretical assumption of a 0% pair-wise correlation and ignoring the uncertainty of the recovery rates: all obligors would default purely according to their marginal probability and the joint distribution of defaults can be derived from the convolution of the individual density functions. The other extreme case of a theoretical 100% pair-wise correlation means that the same default factor realization is applied to all obligors. Hence all obligors with a certain probability or higher will default and none of the others will. The P&L distribution in the 100% correlation case reflects the sum of all obligors' PVs for a given default probability. For example, the 0.1th (99.9th) percentile of the P&L (loss) distribution is equal to sum of P&Ls from defaults for all obligors with an individual default probability of 0.1% or higher. In a long/short portfolio, it is possible that this value is zero because of a "dominance" of short exposures to the defaulting names. The DRC could hence be *lower* (in absolute terms) for a *higher* pair-wise correlation. Particularly, also monotonicity of the DRC for in-between cases with correlations between 0% and 100% correlation is not ensured. As pointed out in Laurent *et al.* (2016) the required number of factors in the correlation model is usually portfolio-dependent: in a directional portfolio, a one-factor approximation will tend to perform quite well, while for more realistic long/short portfolios even two factors are hardly able to capture the dependence structure.

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8 Only downward changes in correlation are considered given that the average correlation in the model is already at the highest level over the considered time period.

As an example of a more structural model parameterization risk the sensitivity of the DRC to the systematic factor weight in the recovery rate model ρ^Y is analyzed as case (d). A change in this parameter from 4% to 25% increases the DRC for all portfolios. This is expected given that an increase in ρ^Y has the effect of decreasing the average conditional recovery rate for extreme scenarios. In these scenarios, which are driven by very negative global asset returns, the average recovery rate across the defaulted names will be monotonically lower for higher values of ρ^Y. For example, taking the most extreme simulated scenario for the long-only *investment-grade portfolio A* – corresponding to 75 defaults out of 125 names – the average conditional recovery rate amounts to about 34% for ρ^Y = 4% and to about 23% for ρ^Y = 25%. These values are significantly lower than the marginal average conditional recovery rate of 45%, reflecting the depressed recoveries in such an extreme scenario. For long-only *portfolios A, C and E*, the DRC increase due to the increase in ρ^Y is in the range of 3% to 11%. For long-short *portfolios B, D and F*, the increase is around 2% to 3% while for the aggregate *portfolios G and J*, the increase is around 5%.

Notably, the presented analyses are conducted ceteris paribus and certain "cross effects", for example, between ratings (default probabilities) and correlations, might be worth monitoring in a practical application.

Table G.1:
Selected sensitivity analyses for DRC figures

This table presents the results of selected sensitivity analysis of the DRC figures relative to the base cases in Table 4 in the main article; a positive number indicates a more negative P&L (i.e. an increase in DRC). The stresses to the average recovery rate are achieved by adjusting μ^{RR} from Table 3 in the main article. The stresses in the average default correlation are achieved by means of adjusting the R^2 of the names in the portfolio while leaving the systematic risk factor structure unchanged. In both cases, the recovery rate model is recalibrated to ensure that the recovery rate distribution conditional on default across all ratings matches the targeted moments (see Table 3 in the main article). The stresses for both average recovery rate and average default correlation are relative changes.

	(a) Obligor ratings		**(b) Average recovery rate**		**(c) Average default correlation**		**(d) Recovery rate: weight of systematic factor**
	Downgrade by one notch	Upgrade by one notch	Decrease by 25%	Increase by 25%	Decrease by 25%	Decrease by 10%	$\rho^Y = 0.25$
A. Investment-grade bonds – long position	33.7%	-23.2%	16.4%	-16.2%	-29.0%	-12.2%	7.7%
B. Investment-grade bonds – long/short position	16.0%	-19.4%	15.4%	-16.1%	-11.8%	-5.1%	3.2%
C. High-yield bonds – long position	23.4%	-21.1%	16.0%	-15.7%	-18.8%	-7.6%	10.8%
D. High-yield bonds – long/short position	20.4%	-12.8%	15.0%	-15.8%	-6.0%	-2.7%	3.6%
E. Sovereign bonds – long position	34.2%	-15.8%	16.7%	-16.4%	-11.8%	-1.7%	2.9%
F. Sovereign bonds – long/short position	12.4%	-2.7%	11.7%	-13.6%	2.0%	1.0%	1.7%
G. Portfolios B, D and F together	*13.4%*	*-10.9%*	*15.4%*	*-15.7%*	*-5.7%*	*-2.6%*	*4.4%*
H. Equity index constituents – long position	20.0%	-20.0%	N/A	N/A	-40.0%	-20.0%	N/A
I. Equity index constituents – long/short position	0.0%	0.0%	N/A	N/A	0.0%	0.0%	N/A
J. Portfolios B, D, F and I together	*14.4%*	*-11.0%*	*14.0%*	*-13.4%*	*-6.9%*	*-2.7%*	*4.6%*

References

Basel Committee on Banking Supervision (2016), Minimum Capital Requirements for Market Risk, January.

Du, Wenxin; Schreger, Jesse (2013), Local Currency Sovereign Risk, Working Paper, Board of Governors of the Federal Reserve System, December.

Dunn, Gary (2008), A Multi-period Gaussian Copula Model of Default Risk, Working Paper, UK Financial Services Authority, August.

Laurent, Jean-Paul; Sestier, Michael; Thomas, Stéphane (2016), Trading Book and Credit Risk: How Fundamental is the Basel Review?, *Journal of Banking and Finance*, Vol. 73, pp. 211-223.

Longstaff, Francis A.; Pan, Jun; Pedersen, Lasse H.; Singleton, Kenneth J. (2011), How Sovereign is Sovereign Credit Risk?, *American Economic Journal: Macroeconomics*, Vol. 3, pp. 75-103.

Moody's (2011), Sovereign Default and Recovery Rates, 1983-2010, Special Comment, May.

Smillie, Alan; Epperlein, Eduardo (2007), Basel II: Compliance by Default, *Risk*, Vol. 20, December, pp. 96-100.