The Royal Bank of Scotland

Controlling volatility to reduce uncertainty

The past two years have seen a reduction in risk appetite from investors, with clients reverting to less complex payoffs. However, while payoff variety has contracted, creation of new underlying indexes has proliferated. Most notably, a new breed of transparent rule-based indexes, known as 'dynamic strategies', has become very popular as their inherent adaptability may help investors navigate through challenging market conditions

Volatility controlled options

Investors are typically attracted to capital-protected products because they have positive personal expectations of the underlying assets' future returns. These products are typically structured with capital protection being provided through investment of the present value to be protected in a zero coupon bond and positive exposure to the underlying asset achieved through purchase of a call option, which may be a vanilla option or something more bespoke. Regardless of the type of option being used, its price will be heavily influenced by either the volatility of the underlying asset or the distribution of its returns, both of which are stochastic.

However, volatility control (VC) seeks to eliminate this source of uncertainty and target an *a priori* level of selected volatility. VC is implemented by dynamically adjusting the investor's exposure to an underlying reference index. Typically, this occurs daily and exposure ideally is a function of both target volatility (TV) and future volatility (FV), such that *exposure = TV/FV*. Because FV is unknown, various proxies can be used, including implied volatility or statistical forecasting methods, but for the sake of simplicity and transparency, most often recent historical realised volatility (RV) is used in place of FV.

Attractive features of VC are luring both option buyers and sellers

Heteroscedasticity of future returns is inevitable in stock markets as volatility can be dramatically impacted by different market regimes. This uncertainty creates problems for both buyers and sellers of options, which can be remedied through application of a VC overlay. Option buyers find that their mark-to-market valuations are often affected as much by changes in volatility (which they usually have no view on) as they are to changes in the underlying asset (which they are solely interested in). The dynamic control that the VC overlay gives attempts to stabilise and normalise the future variance of the distribution of returns. Equity returns exhibit a distribution, which is negatively skewed, and VC is effectively adapting exposure to the different market modes. It provides higher exposure in positive market cycles, which are characterised by lower volatility and automatically shields investors via lower exposure during negative market phases that are categorised by a high-volatility regime. This level of control means that the price of options can be tailored by setting a desired level of TV and by prescribing how the participation is to be varied. In addition, investment returns may be enhanced by virtue of the dynamic exposure mechanism, in particular by automatically deleveraging exposure to the equity underlying when markets experience corrections.

The S&P 500 has returned 6.8% per annum since the end of 1950. It suffered 11 bear markets, which lasted, on average, 15 months each, and returned -35.8% annualised at realised volatility (RV) of 22.0%. By contrast, the intermittent bull markets lasted, on average, 51 months each and returned 21.4% annualised at RV of only 13.7%. We have, in reality, at any time one of two extremely different distributions that prevails. As shown in figure 1, the overall distribution is negatively skewed, though bull market skew is positive with excess kurtosis, suggesting a positive tail. Despite accounting for the minority of observations (21% of the time spent in bear territory), the bear market skew and kurtosis dominate the overall distribution, whereas mean and volatility more closely resemble that of the bull market distribution. The bull markets witnessed historically thus compound over long periods of time (four years, on average) at low volatility, suggesting they can be captured gradually. The bear market tails happen quickly. A short-term, conservative RV measure within the

VC then allows full exposure to bull market tails, while reducing exposure to the bear market tails. This ability to adapt offers a huge advantage over more traditional structured products, which maintain a fixed exposure.

The risk of extreme events is manifested in the empirically observed implied volatility skew. Since equity markets more often gap down than up, out-of-the money puts are more expensive than out-of-the money calls. Option sellers, like any insurer, are essentially charging a premium to cover these events. The VC attempts to mitigate these tail events, reducing risk for the seller and hence the cost to the buyer. The VC allows the buyer and seller to set the price between them by greatly reducing risks neither of them want exposure to.

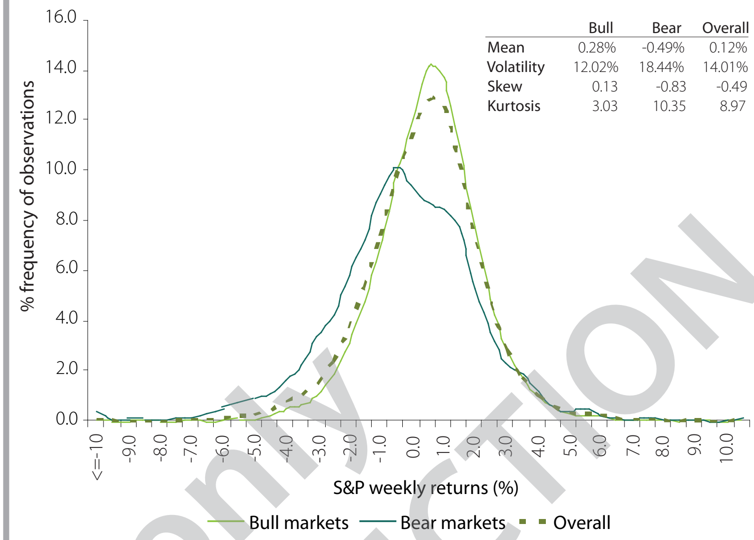
Pricing analysis of VC options

As we've seen, VC achieves an outcome that is beneficial to both seller and buyer. In the absence of vega, the actual pricing of options becomes greatly simplified, as many of the other inputs into options pricing models are fixed or totally hedgeable. A simple Black-Scholes model may seem like the obvious choice to price these options but care should be taken when pricing even vanilla calls on a VC underlying. To illustrate why, we consider here three different dynamics for the S&P 500, apply VC and look at how it affects the prices of three-year European call options for a range of strikes. We calibrate all models to the same S&P-implied volatility surface. The first model uses Dupire local volatility in the stochastic diffusion, the second adds a Merton jump process to the local volatility model and the third uses a Heston stochastic volatility model. A target volatility of 15% is used and rebalancing occurs daily according to *participation = TV/RV*. Call option prices are computed and the implied volatilities backed out.

Figure 2 plots the implied volatilities as a function of strike. What is immediately clear is that the implied volatilities exhibit skew. With the underlying volatility maintained at the preset target level, that may seem counterintuitive. The skew is a consequence of the negative spot volatility correlation that is manifest in the S&P implied volatility surface. The skew seen here is coming from the fact that, as spot goes down and volatility goes up, the participation will be reduced, which is beneficial to the option holder and hence increases the price. The VC underlying will subsequently be participating less in future downwards moves. The second feature to observe from figure 2 is that stochastic volatility is less expensive than the local volatility without jumps, which is less expensive than the local volatility with jumps. Intuition here would suggest that – in the presence of the observed negative skew in the S&P implied volatility surface – the stochastic volatility model would be cheaper than the local volatility model. In the stochastic volatility model, as spot goes down, the instantaneous volatility will generally increase, and this in turn will lead to reduced participation. In the local volatility model, as spot goes down, the volatility will increase and in turn lead to a reduced participation also. The difference here is in the degree of certainty about the spot-volatility relationship. With positive skew, the relationship would switch and the stochastic volatility price becomes more expensive than the local volatility one. Finally, the introduction of jumps increases the price further still, as the positive gamma of the call ensures these jumps have a positive effect on the price.

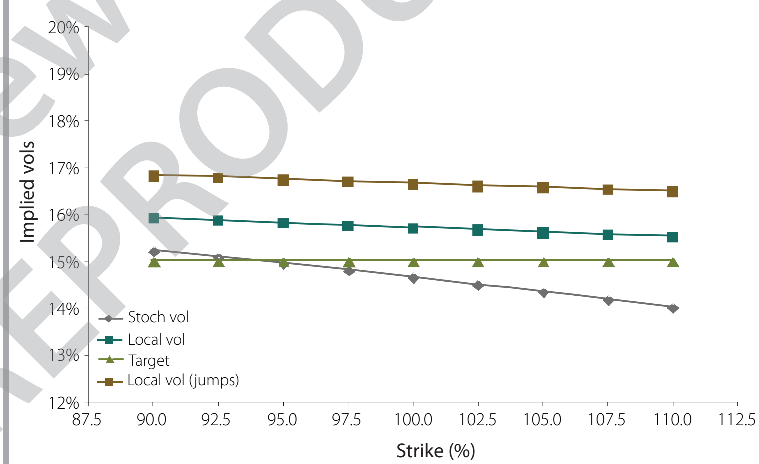
It is interesting to note the observed skew and model dependence of VC options. European call options are about as simple an option as possible and yet we have seen that there are subtle considerations that must be made when pricing them on a VC underlying. As more complicated payoffs and different underlying asset classes (foreign exchange and interest rates, for example) are used in VC products, new and interesting features will inevitably present themselves.

1 Historic S&P return distributions by market environment



	Bull	Bear	Overall
Mean	0.28%	-0.49%	0.12%
Volatility	12.02%	18.44%	14.01%
Skew	0.13	-0.83	-0.49
Kurtosis	3.03	10.35	8.97

2 Plot of implied vols as a function of strike



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