Collateralised exposure modelling: bridging the gap risk

Market-driven defaults, such as that of Archegos, point to the importance of wrong-way risk, concentration and leverage in shaping the tail of the credit loss distribution. Here, Fabrizio Anfuso presents a minimal framework for the joint dynamics of the market risk factors, the trade and collateral portfolio and the overall balance sheet of the defaulting counterparty. Based on this framework, directly applied to the relevant example of an equity swap, the author draws general conclusions that can be used to improve the risk sensitivity of existing exposure metrics, especially in the presence of concentration, leverage and excess collateral

In the wake of the Archegos default (Credit Suisse 2021), there has been consensus in listing the usual suspects (wrong-way risk (WWR), concentration and leverage) as the most dangerous triggers of gap risk (defined as the risk of sudden jumps in portfolio exposure), particularly for supposedly over-collateralised portfolios.

In this paper, we consider the structural WWR introduced for collateralised counterparties (CPTYs) by the contractually agreed process of margining, where the daily rebalancing of the portfolio performance by means of exchanging variation margin (VM) and initial margin (IM) may significantly affect the CPTY's exposure at default (EAD). Indeed, we show that counterparty credit risk (CCR) exposure may be significant even in the presence of IM, and we discuss the potential implications for credit risk management, valuation adjustments (XVA) and the internal model method (IMM).

This is particularly relevant in the context of the prime brokerage business, where clients desire leverage and often run directional and concentrated portfolios, but brokers rely heavily on margins to risk-manage such portfolios.

The aim of our analysis is to understand under what conditions (of portfolio concentration, leverage and market correlations) margin-triggered defaults can cause a collateral shortfall materially beyond the magnitude estimated with the Basel margin period of risk (MPOR) framework. To do so, we focus on the concrete example of an equity swap portfolio and we reconsider *ab initio* the calculation of the CCR exposure in the presence of a collateral agreement, showing the insufficiency of the standard approaches to exposure measurement for concentrated and leveraged counterparties.

Our results are directly applicable to concentrated equity swaps portfolios and, at least on a qualitative level, to similarly leveraged and concentrated positions. In the following, we outline how a solution based on precomputed add-ons can potentially be used to account for collateral shortfalls in existing CCR exposure implementations.

From a technical perspective, two main ingredients are introduced: a 'microscopic' model for the joint dynamics of the CPTY balance sheet and the portfolio underlying assets, and a stochastic process that allows for discontinuous moves in the portfolio underlying assets.

Dickinson (2022) showed how Levy processes can be used to quantify the credit risk of leveraged exposures. Here, we rely on a similar set of tools to introduce a balance sheet model and describe coherently the interplay between CPTY characteristics (such as probability of default (PD) and equity capital), portfolio features (such as WWR, concentration and collateral) and market risk dynamics, including jumps.

The model

Description. Consider two CPTYs, A and B, entering a derivative contract, short and long, respectively, on a single equity position. In the following, our aim is to compute the loss distribution of the surviving CPTY A conditional upon B's default.

To do so, we introduce a model that describes the correlated dynamics of the A-B contract fair value and B's overall balance sheet (denoted by Θ).

The A-B relation and the balance sheet can be characterised as follows:

(1) The derivative synthetically tracks the equity performance. The A-B portfolio can be thought of as a total return swap (TRS) on a single underlying S, where the TRS is at-the-money (ATM) at $t = 0$ and the interest rate leg is ignored. The maximum loss from B's perspective is equal to the starting value of the underlying, $S(0)$, multiplied by the notional of the contract. Throughout the paper, we absorb the contract notional in the definition of S and $S(0)$.

(2) The contractual terms assumed are (i) daily bilateral VM to fully offset the current mark-to-market (MtM) value of the trade; (ii) unilateral (B posting only) $\mathrm{IM} = \alpha S$.[1]

(3) The balance sheet size Θ is shown in figure 1, where the equity capital is referred to here as the net asset value (NAV). In our stylised model: (i) Θ is stochastic with constant liabilities L; (ii) the nominal PD at the one-year (1y) horizon is considered as an input;[2] (iii) the Θ's probability distribution at the one-year horizon is known and depends on the starting value $\Theta(0)$, as well as on the calibration of the correspondent stochastic differential equation (SDE) (see (4)).

(4) Before entering the TRS, L marks the threshold for Θ that triggers B's default. Therefore, we consider $\Theta(0)$ as given and we compute the liabilities as:

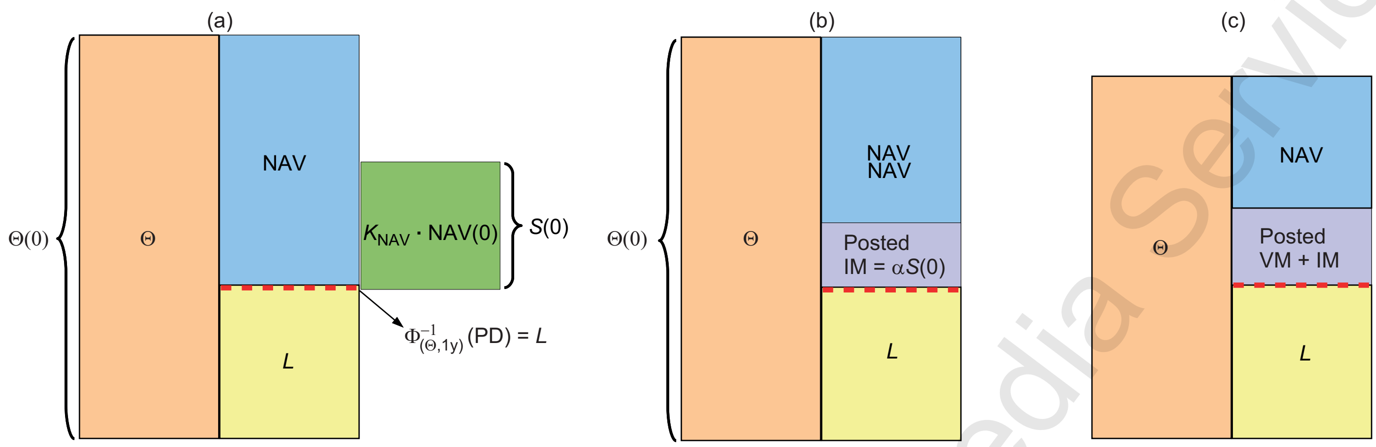
$$L = \Phi^{-1}_{(\Theta,1\mathrm{y})}(\mathrm{PD})$$

(implicitly defining the pre-TRS net asset value, $\mathrm{NAV}(0) = \Theta(0) - L$).

[1] *Here, α is a constant. It can be set (see our example in figure 3) as a tail quantile of the S model distribution at the MPOR horizon, ie, $\alpha = 1 - (\Phi^{-1}_{(S,2w)}(q = 0.01))/S(0)$. The* explicit *leverage of the position is defined as $1/\alpha = S(0)/IM$.*

[2] *Throughout the paper, we always refer to the nominal PD (ie, the probability of default as assessed before entering the TRS trade). In practice, for the extreme leveraged cases, the actual PD may differ significantly from the nominal one (with no implications for our findings).*

1 The CPTY B's balance sheet, as outlined in the model description



The three panels show: (a) $t = 0$, pre-TRS contract; (b) $t = 0$, post TRS (ATM) contract; (c) $t > 0$, one possible stochastic scenario where the balance sheet size Θ and the NAV decrease while the posted collateral amount increases ($S(t) < S(0)$)

(5) After entering the TRS, the balance sheet includes VM and IM where, for convenience, posted collateral obligations are carved out from the NAV definition. And hence (i) VM tracks the performance of the TRS (ie, it can be either posted collateral or part of the NAV); (ii) IM is always part of the posted collateral until contract maturity (although in varying amounts); (iii) the default time of B is now identified as the first $t > 0$ such that $\text{NAV}(t) < 0$ (ie, B is unable to meet its pre-existing liabilities or a new margin call).

(6) In our notation, $S(0)$ is expressed as a fraction of the $t = 0$ NAV (pre-TRS) via the constant K_{NAV}, so as to highlight the 'implicit leverage' of the position.[3] K_{NAV} gives an account of the additional risk (in proportional terms) for B from the TRS portfolio. And it provides an indicative measure of how much a drop in the value of S is likely to affect the PD of B.

Based on the above, the joint dynamics of S and Θ is described by the following system of SDEs:

$$\begin{aligned}
\mathrm{d}S(t) &= \tilde{\mu}_S S(t)\,\mathrm{d}t + \sigma_S S(t)\,\mathrm{d}W_S(t) + \beta_S S(t)\,\mathrm{d}J_S(t) \\
S(0) &= K_{\text{NAV}} \cdot \text{NAV}(0) && (1) \\
\Delta_{\text{mf}}(C_{\text{VM}}(t)) &= -\Delta_{\text{mf}}(S(t)), \qquad C_{\text{VM}}(0) = 0 && (2) \\
\Delta_{\text{mf}}(C_{\text{IM}}(t)) &= \alpha\Delta_{\text{mf}}(S(t)), \qquad C_{\text{IM}}(0) = \alpha S(0) && (3) \\
\mathrm{d}\Theta(t) &= \tilde{\mu}_\Theta \Theta(t)\,\mathrm{d}t + \sigma_\Theta \Theta(t)\,\mathrm{d}W_\Theta(t) \\
&\quad + \beta_\Theta(1-\beta_S\rho_J)\Theta(t)\,\mathrm{d}J_\Theta(t) && (4) \\
&\quad + \beta_S\rho_J\Theta(t)\,\mathrm{d}J_S(t) \\
\mathrm{dNAV}(t) &= \mathrm{d}\Theta(t) + (1-\alpha)\Delta_{\text{mf}}(S(t)) \\
\text{NAV}(0) &= \Theta(0) - L - \alpha S(0) && (5)
\end{aligned}$$

where:

- the S and Θ dynamics are described on an individual basis by the Merton jump-diffusion model;
- W_S and W_Θ are the continuous Brownian components of the S and Θ geometric dynamics (their instantaneous correlation is given by $\mathrm{d}W_S(t)\,\mathrm{d}W_\Theta(t) = \rho_W\,\mathrm{d}t$);
- J_S and J_Θ (with intensities λ_S and λ_Θ and normally distributed jumps $\phi(\mu_{j_S}, \sigma_{j_S})$ and $\phi(\mu_{j_\Theta}, \sigma_{j_\Theta})$) are compound Poisson processes providing the discontinuous 'gap risk' components of the S and Θ dynamics;
- the drifts:

$$\tilde{\mu}_\Theta = \mu_\Theta - (\beta_\Theta(1-\beta_S\rho_J)\lambda_\Theta\mu_{j_\Theta} + \beta_S\rho_J\lambda_S\mu_{j_S})$$

and:

$$\tilde{\mu}_S = \mu_S - \beta_S\lambda_S\mu_{j_S}$$

include the compensation for the jump processes;
- $C_{\text{VM}}(t)$ and $C_{\text{IM}}(t)$ indicate the collateral balances from the non-defaulting CPTY A's perspective;
- mf is the contractual margining frequency (one day (1d) in the following) and $\Delta_{\text{mf}}(\cdot)$ indicates the accrued change over the period mf based on end-of-day (EOD) market quotes (in our case, $\Delta_{1\text{d}}C_{\text{VM}}(t) = -\Delta_{1\text{d}}(S(t)) = (-S(t) + S(t-1\text{d}))$, with $S(t)$ and $S(t-1\text{d})$ being the EOD quotes as of t and as of the day before, respectively);
- β_S and β_Θ are binary variables that will be used below to realise different risk configurations; and, finally,
- ρ_J measures how much Θ is directly affected by a sudden variation in the value of S, accounting for any exposure (other than the A-B portfolio) to the same or to strongly correlated underlyings.

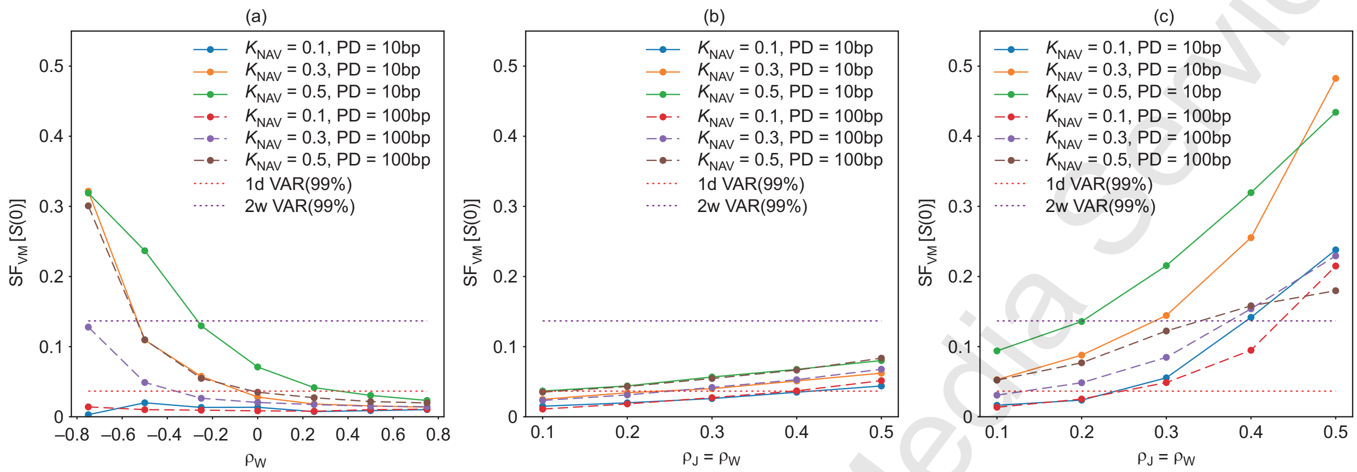
The model calibration and its scope of applicability. Despite its simplicity, the model defined in (1)–(5) already has a large number (16!) of parameters (to account for PD, α, $\Theta(0)$, μ_Θ, σ_Θ, λ_Θ, μ_{j_Θ}, σ_{j_Θ}, K_{NAV}, μ_S, σ_S, λ_S, μ_{j_S}, σ_{j_S}, ρ_W and ρ_J). We shall introduce the following assumptions to reduce the dimensionality:

- $\mu_S = \mu_B = 0$, since any reasonable drift will not affect extreme loss scenarios over a one-year horizon;[4]

[3] *Since the notional of the contract is absorbed in the definition of S and the starting value of the underlying is given, we effectively tune the notional so as to match the desired level of implicit leverage.*

[4] *As a consequence of this choice, both S and Θ are martingales, providing us with $S(0)$ as a natural unit, since $\mathbb{E}[S(t)] = S(0)$.*

2 VM collateral shortfall



(a) Limit case (L3): SF_{VM} in the presence only of S jumps is shown as a function of ρ_W for jump calibration JTD and different levels of the PD and K_{NAV}. (b) Limit case (L4): SF_{VM} in the presence of correlated S and Θ jumps is shown as a function of the jump correlation ρ_J ($\rho_W = \rho_J$) for different levels of the PD and K_{NAV} and jump calibration NDJ. (c) Limit case (L4): SF_{VM} in the presence of correlated S and Θ jumps is shown as a function of the jump correlation ρ_J ($\rho_W = \rho_J$) for different levels of the PD and K_{NAV} and jump calibration JTD. Throughout the figure SF_{VM} is expressed in units of $S(0)$ and the 99th percentile value-at-risk (VaR(99%)) values for the TRS trade over one-day (1d) and two-week (2w) horizons are shown for comparison (also in units of $S(0)$). Note that the IM is often computed as a VaR(99%) metric of the MtM distribution over a 2w horizon

- $\sigma_S = \sigma_\Theta = \sigma_W$ (ie, we assume some risk homogeneity across the assets of CPTY B, at least when it comes to the diffusive components);
- $\sigma_{J_S} = \sigma_{J_\Theta} = 0$ (ie, we focus on constant jumps, so as to keep a simple distinction between the continuous and discontinuous stochastic components);
- $\mu_{J_S} = \mu_{J_\Theta} = \mu_J < 0$ (ie, we introduce the same risk homogeneity assumption as that made for the diffusion volatilities above);
- for the jump parameters $[\lambda_J, \mu_J]$, we consider two example calibrations: near-death jumps (NDJ) (−50%, on average once in 10y) and jump-to-default (JTD) (−99%, on average once in 100y), where the former stylises a large equity down move not leading to the default of the issuer (whereas the latter does lead to a default and occurs with a likelihood commensurate with the PD of the issuer).[5]

Finally, our analysis for the single equity swap case (see below) leverages on the simplicity of the portfolio and on the analytical properties of the model introduced. Nevertheless, in the presence of additional trades (eg, for multiple equity swaps that are also in a long-short combination), we can still use this approach to gauge the collateral shortfall (as defined in the next section). From a calibration perspective, this will require us to:

- identify the most material concentration in the portfolio, considering clusters of correlated assets (eg, from the same sector and/or country),[6] and assign to it a suitable calibration, where the jump size should be approximately scaled down by the relative weight of the overall position;
- estimate ρ_J as an approximate measure of the correlation between the identified concentrated position(s) and the asset side of the balance sheet at $t = 0$.

VM driven defaults and collateral shortfall

In the standard MPOR framework, the default time is defined as the first time t_d that the defaulting counterparty is unable to meet its contractual obligations. And the MPOR horizon is chosen so as to account, in the exposure at closeout $t_c = t_d + \text{MPOR}$, for the risks related to the trade's replacement and collateral liquidation. In practice, this means that the collateralised exposure is computed for a (given) default time as:

$$EE(t_d) = \mathbb{E}[\max\{\text{MtM}(t_c) - C(t_d^+) + \cancel{C(t_d^+) - C(t_d^-)}, 0\}] \tag{6}$$

where, for simplicity, we consider only collateral posted in the settlement currency (ie, carrying no risk over the MPOR), and the cancelled term is ignored.

As discussed more extensively later, the standard approach implicitly assumes that the contractual obligations at t_d^- and t_d^+ are *de facto* equivalent (ie, the collateral shortfall and the exposure, if measured in the limit $\text{MPOR} \to 0$, are generally vanishingly small).

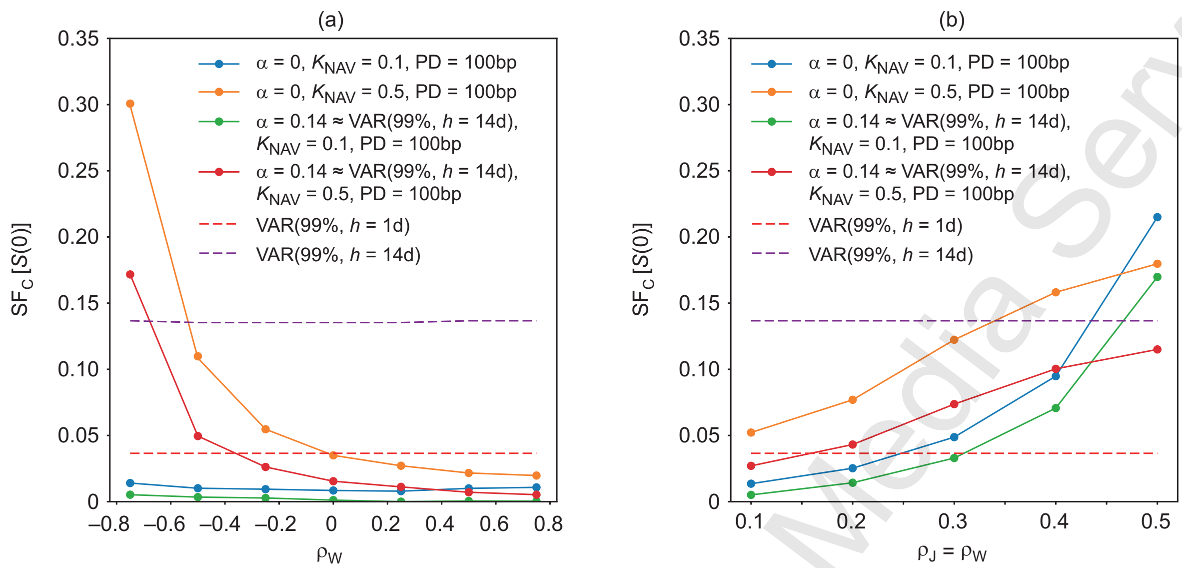
Here, we perform a comprehensive simulation, inclusive of the B credit default event, where the time of default is identified by the first breach of the 'going concern' condition $\text{NAV}(t) \geqslant 0$. As we will see, the collateral shortfall conditional upon default (the cancelled term in (6)) is generally non-zero.

Focussing on VM-only contractual relations first, we define the VM shortfall as the following expectation:

$$\text{SF}_{\text{VM}} = \mathbb{E}[\max\{-\Delta_{1d}(S(t^*)), 0\} \mid \mathcal{D} \neq \emptyset],$$
$$\mathcal{D} = \{t_n \leqslant 1\text{y} \mid \text{NAV}(t_n) < 0\} \tag{7}$$

[5] *To perform a first principles calibration of $[\lambda_J, \mu_J]$, we can consider the frequency of occurrences of equity down moves that qualify as jumps for a large dataset of equity time series. Historically observed jumps can be further bucketed (eg, based on additional dimensions such as country, sector and rating).*
[6] *In the case of Archegos, the long positions on tech shares would have defined such a cluster.*

3 Collateral shortfall in the presence of IM



The impact of the IM on the collateral shortfall ($\alpha \approx 0.14$, corresponding to IM $=$ VaR(99%, 2w)), is shown in panels (a) and (b). SF_C is expressed in units of $S(0)$ and VaR(99%) values over 1d and 2w horizons are shown for comparison (also in units of $S(0)$). (a) Limit case (L5): SF_C in the presence only of S jumps is shown as a function of ρ_W for PD $=$ 100 basis points (bp) and jump calibration JTD. (b) Limit case (L6): SF_C in the presence of correlated S and Θ jumps is shown as a function of the jump correlation ρ_J ($\rho_W = \rho_J$) for PD $=$ 100bp and jump calibration JTD.

where $t^* = \inf(\mathcal{D})$ indicates the first (if any) time on a path when, after (hypothetically) accounting for the margining obligations, the 'going concern' condition is breached.

For the computation of SF_{VM}, we can rely on the model introduced in (1)–(5) to simulate the S and Θ dynamics and detect defaults up to the one-year horizon. SF_{VM} is then obtained as an average of the observed shortfalls (including scenarios where the default is unrelated to the margining process and for which shortfalls are zero).

In the presence of IM, a general definition of the collateral shortfall is less straightforward, primarily because the IM is often held by a third-party custodian (as opposed to VM, which is generally held by the CPTY), and the daily margining process for IM, especially in the case of large VM moves, may depend upon the specifics of the contract. In the following, we assume that the larger of $\mathrm{IM}(t^*)$ and $\mathrm{IM}(t^* - 1\mathrm{d})$ is available to the non-defaulting CPTY to offset the SF_{VM}, ie, the overall collateral shortfall in the presence of IM becomes:

$$\begin{aligned} SF_C &= \mathbb{E}[\max\{-\Delta_{1\mathrm{d}}(S(t^*)) \\ &\qquad - \max\{\mathrm{IM}(t^*), \mathrm{IM}(t^* - 1d)\}, 0\} \mid \mathcal{D} \neq \emptyset], \\ \mathcal{D} &= \{t_n \leqslant 1\mathrm{y} \mid \mathrm{NAV}(t_n) < 0\} \end{aligned} \tag{8}$$

(for example, for the A-B equity swap portfolio, a default event with a material SF_C is driven by a large missed VM margin call partially offset by $\mathrm{IM} = \mathrm{IM}(t^* - 1\mathrm{d})$).

Finally, it is worth emphasising that SF_C materialises before the unwinding of the position and the liquidation of the collateral. Therefore, any additional exposure built up over the MPOR (eg, driven by liquidity and/or market distress) will contribute to the loss; these components (ie, the non-cancelled terms in (6)) are ignored in the following, where we focus on SF_C.

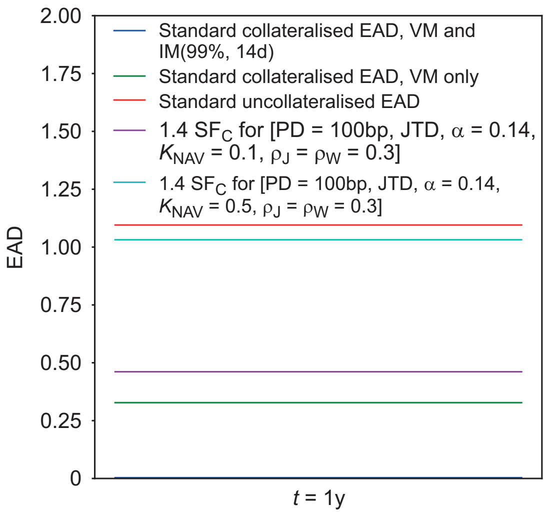
In the next section, we characterise SF_C for the equity swap portfolio as a function of the model parameters, focusing on cuts in the calibration space where gap risk, concentration and leverage materially affect the resulting exposures.

Expected collateral shortfall for the equity swap portfolio

In our analysis, we consider the following limiting cases (see table A for the details of the inputs):

- **(L1/L2) Continuous Θ and S (VM only).** Continuous Θ and S is the baseline case (L1), where we confirm that the SF_{VM} is negligible, consistently with the Basel III MPOR framework. Adding Θ jumps only (L2) has hardly any impact. For both (L1) and (L2), the SF_{VM} is of the order of the daily diffusion volatility ($\approx 0.4\sqrt{\sigma_W/365} \approx 0.006$).
- **(L3) Continuous Θ and discontinuous S (VM only).** The presence of S jumps (see figure 2(a)) significantly increases the materiality of SF_{VM} wrt (L1) and (L2). Note the following:
  - As a function of the implicit leverage K_{NAV}, SF_{VM} can quickly reach the level of the IM and beyond.
  - The shortfall curves are inversely proportional to ρ_W. In practice, the S and Θ WWR dynamics can push the trade's MtM and A's posting requirements up, while depleting B's balance sheet size, and a large shortfall scenario may occur when, after an upside trend for S, a jump event triggers a margin call that cannot be met.
  - The level of the PD does not change the qualitative behaviour but has an impact on the size of SF_{VM}. For lower PDs, the marginal weight of jump-driven defaults is larger.
- **(L4) Discontinuous Θ and S processes (VM only).** A positive ρ_J component of S jumps in the Θ SDE can generate substantial shortfalls also at moderate levels of K_{NAV} (see panels (b) and (c) of figure 2). Note that:

4 Impact of WWR, concentration and leverage on the EAD



EAD for a TRS trade (ATM, $S(0) = 10$). Five cases are considered (from bottom to top): standard collateralised EAD in the presence of IM $=$ VaR(99%, 2w); standard collateralised EAD, VM only; $1.4 \cdot \mathrm{SF_C}$ in the presence of IM $=$ VaR(99%, 2w), computed for [PD $=$ 100bp, JTD, $\alpha = 0.14$, $K_{\mathrm{NAV}} = 0.1$, $\rho_j = \rho_W = 0.3$], where the 1.4 factor is the so-called alpha multiplier as defined in Basel II (Basel Committee on Banking Supervision 2005); $1.4 \cdot \mathrm{SF_C}$ in the presence of IM $=$ VaR(99%, 2w), computed for [PD=100bp, JTD, $\alpha = 0.14$, $K_{\mathrm{NAV}} = 0.5$, $\rho_j = \rho_W = 0.3$]; standard uncollateralised EAD

- ρ_J depletes Θ whenever S experiences a (negative) jump and hence effectively increases the implicit leverage. Something quite similar to this mechanism (which does not need to be perfectly synchronous as stylised here) played a major role in the Archegos default.[7]
- As for (L3), $\mathrm{SF_{VM}}$ can quickly reach the level of (even multiples of) the IM, including for significantly lower levels of K_{NAV} in this case.
- For the (L4) example, the diffusion correlation ρ_W (aligned with ρ_J) is for right-way risk only. WWR among the continuous Brownian drivers increases the level of the shortfalls even further. However, this does not seem to be a commonly realised set-up.
- For large ρ_J, crossings may occur between $\mathrm{SF_{VM}}$ curves with different values for K_{NAV} (see, for example, curves $K_{\mathrm{NAV}} = 0.4$ and $K_{\mathrm{NAV}} = 0.5$ at $\rho_J \approx 0.4$). This effect is driven by the increasing likelihood of continuous defaults for higher implicit leverages, where the jumps are instrumental in bringing Θ closer to the default line, but the actual default is triggered by a smooth random change.

(L5/L6) Including IM. The presence of excess collateral (of the order of standard initial margin model (Simm) IM (Isda 2021)) does reduce the impact of the implicit leverage on $\mathrm{SF_C}$ both for the S jumps only ((L5), figure 3(a)) and for the S and Θ jumps cases ((L6), see panel (b) of figure 3 and figure 4).

This reduction is mostly driven by the increased likelihood of continuous defaults, since the IM practically counts as an additional liability. If the default is not smooth, $\mathrm{SF_C}$ is not greatly affected by the excess collateral, primarily because of the simplicity of the model considered, where jumps can have only one size (for both JTD and NDJ, $\mu_J \gg \alpha$). Using a more realistic jump dynamics would allow us to better resolve the loss-absorbing effect on the IM for discontinuous defaults too.

A. Model calibrations used for the different limiting cases (L1)–(L6).

Case	PD	α	β_S	β_Θ	$\Theta(0)$	σ_W	λ_J, μ_J	K_{NAV}	ρ_W	ρ_J
L1	×	0	0	0	10	0.3	0	×	×	0
L2	×	0	0	1	10	0.3	JTD	×	×	0
L3	×	0	1	0	10	0.3	JTD	×	×	0
L4	×	0	1	0	10	0.3	×	×	ρ_J	×
L5	100bp	×	1	0	10	0.3	JTD	×	×	0
L6	100bp	×	1	0	10	0.3	JTD	×	ρ_J	×

An '×' entry indicates that multiple values are considered for the given parameter; another parameter as entry (eg, (L4), ρ_W) indicates that multiple values are considered in sync with the entry parameter (ρ_J).

Making EADs and PFEs non-zero …

Above, we showed how collateral shortfalls can be comparable with or even larger than the typical IM. This is directly relevant for the EAD and potential future exposure (PFE) of, for example, funds and hedge funds portfolios (Cesa 2022), and more generally for all those CPTYs with high levels of concentration and implicit leverage.[8]

As shown in figure 4, the direct application of the standard path-wise collateral modelling used in IMM and XVA (Gibson 2005) produces EADs of approximately zero whenever an IM of the order of Simm (Isda 2021) is present (making the portfolio effectively risk-free from both a CCR Pillar 1 perspective and a risk monitoring perspective).

This is not surprising, since the industry standard approach does not contemplate any shortfall at default, and the portfolio exposure is built up by random increments over the MPOR generated by risk factors evolution (RFE) models that are typically continuous and do not account for WWR endogenously.[9] Since the simulated exposures will exceed the VM $+$ IM only for a tiny fraction of scenarios, the final EAD and PFE results are often close to zero.

Converting established modelling frameworks to support discontinuous dynamics *ab initio* presents major challenges, including the additional model risk of the enhanced calibration. Nevertheless, an alternative approach worth considering is modifying the standard collateral model to account for collateral shortfalls exogenously. For example, introducing a framework along the following lines:

- Identify the main portfolio concentrations (see also the model calibration section) and their risk profiles. This can be done in multiple ways, either statically (based on the $t = 0$ portfolio) or dynamically (at scenario level, in simulation). The former approach can be readily implemented (eg, relying on the standardised approach for counterparty credit risk (SA-CCR) (Basel

[7] *Here we refer to the domino effect of tech shares down-moves that was initiated by the major drop in the value of Viacom ($\approx -54\%$).*

[8] *Archegos is an obvious example, with up to 85% of the NAV tied up as collateral and the quasi-totality of the assets invested long in a group of highly correlated tech shares.*

[9] *Pykhtin (2012) and Andersen* et al *(2017) suggest material improvements to this oversimplified picture, but still rely primarily on continuous RFEs. Some practical extensions to discontinuous RFEs have been considered by Andersen & Dickinson (2019).*

Committee on Banking Supervision 2014) hedging sets' PFEs). The latter offers the possibility of detecting scenario-dependent concentrations.

- Assess the counterparty implicit leverage K_{NAV},[10] referring to the main portfolio concentrations and, for example, based on stress testing of the $t = 0$ portfolio and information such as the NAV.
- Gauge the level of the correlations ρ_J and ρ_W (see also the model calibration section) between the concentrated portfolio underlyings and the counterparty assets.
- Include SF_{VM} add-ons in the exposure calculation (this can be a static or a dynamic amount depending on how concentration is measured). Those add-ons can be precomputed for a broad range of risk factors, payouts and calibrations, and associated with the portfolio (or portfolio scenarios) based on leverage, concentration and correlations.

In general, we expect that, for concentrated and leveraged positions, SF_{VM} will offset a large part of the IM, thus restoring EAD and PFE as non-zero measures of risk. Notice in this regard that accounting for VM shortfalls is the primary reason to introduce IM anyway. *Hence, not considering the effect of gap moves and potential collateral shortfalls at default in the exposure calculation, at least for concentrated portfolios, is tantamount to double-counting collateral.*

[10] *Sourcing K_{NAV} can be challenging for some CPTYs. Nevertheless, a framework to estimate this (or a similar) quantity, either accurately or conservatively, should be an integral part of the due diligence process (given its determining impact on the risk of the transaction).*

Conclusions

We introduced a model for the joint dynamics of the defaulting CPTY balance sheet and of the underlying assets in the portfolio. Based on this, we showed how the interplay of concentration, leverage and correlations may materially affect the exposure of a collateralised equity swap portfolio. In addition, we provided a detailed account of the different emerging risk profiles as a function of the key portfolio's and CPTY's characteristics. Finally, we outlined how, at least conceptually, our findings can be tailored to adjust existing CCR EAD and PFE metrics via a framework based upon precomputed add-ons. ■

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