APPENDIX 1: ANOMALY DETECTION

To determine whether any observations may be considered as anomalies, we make use of the Euclidean distance of each observation from its respective centroid. Intuitively, the further away an observation is (or the greater the distance), the higher confidence we have that it is an anomaly. There are several ways to set the threshold at which an observation becomes an anomaly. In our case, we normalize the distances by converting each value into a modified z-score using the following formula:

$$z_{mod} = \frac{D_i - \text{Med}(D)}{\text{MAD}(D)} \times \Phi^{-1}\left(\frac{3}{4}\right)$$

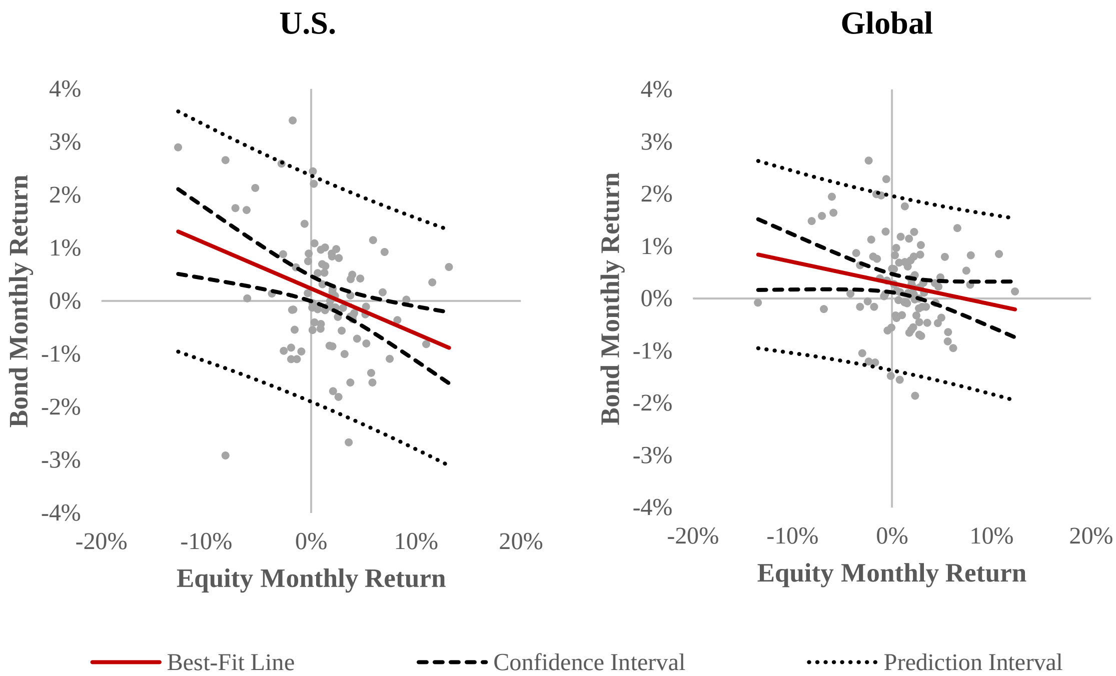
where D_i is the distance of the i-th observation, D is the vector of all distances for a given cluster, Med is the median operator, MAD is the median absolute deviation, and Φ^{-1} is the inverse of the cumulative distribution function for the standard normal distribution. The last of these terms is used as a scaling factor to make the magnitude of the modified z-score comparable to the traditional one (Iglewicz and Hoaglin, 1993). We define an anomaly as any point which has a modified z-score greater than 4, a value which provides a good balance between capturing clear anomalous observations and those which are simply uncommon.

APPENDIX 2: LINEAR REGRESSION

Compared to k-means clustering, an alternative and more standard approach to analyzing the relationship between equity and bond returns would be to use a linear regression. In **Figure A-1** we show the U.S. and global results if we run an ordinary least squares approach. As would be expected, for both scenarios we observe that the “best-fit” line has a negative slope, suggesting a negative relationship between the two asset classes. The black dashed line (“Confidence Interval”) can be thought of as the area in which the true underlying best-fit line is likely to lie within at a 95% confidence level. We see that even at the lower bound, this is likely to remain above zero even when equity returns are negative. In other words, during equity market downturns, we would predict the expected corresponding bond return to be positive.

The dotted black line (“Prediction Interval”) is, for any given equity return, the range that the bond return is highly likely to fall within at a 95% confidence level. In this setting, observations outside of this interval may be considered as anomalies. However, as mentioned in the main text, such a regression-based approach results in low R-squared values and therefore provides a low degree of confidence and investment interpretation. For reference, we also display the standard regression statistics in **Figure A-2**.

Figure A-1. Figures for linear regression of government bond returns on equity returns, October 2000 to March 2021



Notes: The confidence threshold was defined as 5% for both the confidence and prediction intervals. In the case of the confidence interval, this means we would expect the true population regression coefficients to lie within this range with 95% probability. For the prediction interval, we would expect 95% of new observations to lie within the interval. U.S. equity returns refer to the MSCI USA Total Return Index and U.S. bond returns refer to the Bloomberg U.S. Treasury Total Return Index. The yield threshold of 2.5% was based off the U.S. 10-Year Treasury Bond. Global equity returns refer to the MSCI All Country World Total Return Index and global bond returns refer to the Bloomberg Global Treasury Total Return Index. The yield threshold of 1.5% was based off the Bloomberg Global Treasury Total Return Index. Bond returns are hedged to USD and both figures are in USD. Source: Bloomberg L.P., using monthly data from October 2000 to March 2021. **Past performance is no guarantee of future returns. The performance of an index is not an exact representation of any particular investment, as you cannot invest directly in an index.**

Figure A-2. Statistics for linear regression of government bond returns on equity returns, October 2000 to March 2021

U.S.			**Global**		
Constant	0.002		Constant	0.003	**
	(0.001)			*(0.001)*	
Equity	-0.085	**	Equity	-0.041	
	(0.027)			*(0.023)*	
Degrees of Freedom	88		Degrees of Freedom	95	
Standard Error of Regression	0.011		Standard Error of Regression	0.008	
R-Squared	9.9%		R-Squared	3.3%	
Adj. R-Squared	8.9%		Adj. R-Squared	2.2%	
F-statistic	9.695		F-statistic	3.207	

Notes: ** p-value < 0.01, * p-value < 0.05. Numbers in brackets denote the standard errors associated with each regressor. U.S. equity returns refer to the MSCI USA Total Return Index and U.S. bond returns refer to the Bloomberg U.S. Treasury Total Return Index. The yield threshold of 2.5% was based off the U.S. 10-Year Treasury Bond. Global equity returns refer to the MSCI All Country World Total Return Index and global bond returns refer to the Bloomberg Global Treasury Total Return Index. The yield threshold of 1.5% was based off the Bloomberg Global Treasury Total Return Index. Bond returns are hedged to USD and both figures are in USD.
Source: Bloomberg L.P., using monthly data from October 2000 to March 2021. **Past performance is no guarantee of future returns. The performance of an index is not an exact representation of any particular investment, as you cannot invest directly in an index.**

APPENDIX 3: HOW TO DETERMINE THE RIGHT NUMBER OF CLUSTERS

We use two approaches to determine the optimal number of clusters in our analysis. The first technique is the so-called "elbow" chart, in which the within-cluster sum of squared distances (WCSS) is plotted. The WCSS is relevant because this is the objective that the clustering algorithm minimizes. The WCSS is then computed as finding the sum of squared distances between each observation and its associated centroid:

$$WCSS = \sum_{i=1}^{M} (x_i - c_i)^2$$

where x_i is the i-th observation, c_i is the centroid associated with the i-th observation, and M is the number of observations.

Ideally, we want the WCSS to be as low as possible; however, by increasing the number of clusters the WCSS will almost always decrease. Therefore, there is a trade-off between explaining more of the variation and preventing overfitting. An "elbow" is looked for in the curve because for more clusters than this point, a relatively small reduction is achieved in WCSS for each additional cluster. The optimal number of clusters is at the point where the kink occurs.

The second technique is the "Silhouette score":

$$Silhouette\ Score = \frac{b - a}{\max(a, b)}$$

where a is the mean intra-cluster distance for each sample (i.e., the average distance to the other observations in the same cluster) and b is the mean nearest-cluster distance for each sample (i.e., the average distance to observations in the next closest cluster).

When the Silhouette score is high, the observation is well-matched to its own cluster and poorly matched to neighboring clusters, therefore a higher score is better. **Figure A-3** shows the elbow and Silhouette charts for our analysis of U.S. equity and U.S. treasury returns and **Figure A-4** refers to global equity and government bond returns.

Figure A-3. Testing the value of *k* for *k*-means clustering run on U.S. government bond returns and U.S. equity returns, October 2000 to March 2021

Notes: Equity returns refer to the MSCI USA Total Return Index and bond returns refer to the Bloomberg U.S. Treasury Total Return Index. The yield threshold of 2.5% was based off the U.S. 10 Year Treasury Bond. All figures are in USD.
Source: Bloomberg L.P., using monthly data from October 2000 to March 2021.

Figure A-4. Testing the value of *k* for *k*-means clustering run on global government bond returns and global equity returns, October 2000 to March 2021

Notes: Equity returns refer to the MSCI All Country World Total Return Index and bond returns refer to the Bloomberg Global Treasury Total Return Index. The yield threshold of 1.5% was based off the Bloomberg Global Treasury Total Return Index. All figures are in USD and bond returns are hedged to USD.
Source: Bloomberg L.P., using monthly data from October 2000 to March 2021.