In the balance redux

Understanding the interaction between a new derivative contract, its financing and the wider balance sheet during pricing is critical for dealer profitability. For this purpose, Mats Kjaer develops a structural balance sheet model and demonstrates how it can be used to derive consistent firm and shareholder breakeven prices. The latter price often involves a significant capital valuation adjustment, which can be managed statically or hedged dynamically

Incorporating the effects of derivatives funding into its pricing in the form of funding and margin valuation adjustments has become standard practice over the last couple of years. One popular conceptual foundation used to derive these adjustments is the semi-replication, developed by Burgard & Kjaer (2011a, 2013, 2017), which combines bilateral counterparty risk and funding in a consistent way.

Increasing regulatory capital requirements since 2010 have led to a need for capital valuation adjustments to ensure derivatives desks are meeting their return on equity (ROE) targets. Green *et al* (2014) have perhaps the first paper on this topic, in which they extend the semi-replication approach of Burgard & Kjaer (2013) by adding a capital account accumulating at an exogenous ROE. In essence, semi-replication is a credit-hybrid extension of the Black-Scholes-Merton approach; as such, it ignores any balance sheet feedback from derivative cashflows or their financing, including regulatory equity capital. Green *et al* (2014) suggest that 'fully understand[ing] the interrelationship between counterparty default and capital requires a full balance sheet model'. Burgard & Kjaer (2011b), from whose article the title of this one was derived, took feasibly the first step in this direction, but it was Andersen *et al* (2019) who developed this approach much further with their corporate finance-inspired single-period dealer balance sheet model. Their approach is very transparent, allowing them to derive the firm and shareholder breakeven prices of a new derivative contract, albeit without incorporating regulatory capital, equity financing or hedging.

This article amalgamates the works of Kjaer (2017), which extends that of Andersen *et al* (2019) by adding these missing features, and Kjaer (2018), which extends it further to incorporate continuous time. All single-period model intuition carries over to the continuous-time setting, which is much more involved. Hence, we will initially focus on the single-period setup before summarising the corresponding continuous-time results. Even in a single-period setup the full balance sheet model is too large to have practical pricing calculations performed on it. So, we reduce it by deriving marginal firm and shareholder breakeven prices in the small trade limit. The resulting shareholder valuation adjustment formulas are financing method dependent, are valid for arbitrary market and default models, and can be managed statically or by dynamic hedging.

Dealer balance sheet and cashflow statement

To include the effects of regulatory capital and equity funding in pricing, we follow the single-period balance sheet approach of Kjaer (2017). The notation is summarised in table A, where all T_1 quantities are square integrable random variables on the probability space $(\Omega, \mathcal{F}, P)$ and the T_1 legacy assets and liabilities A and L have jointly continuous density functions.

At T_0, a dealer firm is created by shareholders and creditors and endowed with A_0 assets and L_0 liabilities. It then considers entering into a derivative contract with contractual payoff X_c per unit with a counterparty firm. The price $U(q)$ for q units is financed with a mix of debt, equity and/or off-balance sheet cash. The regulator requires the amount of equity financing $U_K(q)$ to exceed some regulatory capital requirement $K(q)$.

We refer to the triplet $(U_S(q), U_F(q), U_K(q))$ as a financing strategy for $U(q)$ in reference to the funding strategies introduced by Burgard & Kjaer (2013). Such a strategy must satisfy the financing constraint:

$$U(q) = U_S(q) + U_F(q) + U_K(q) \tag{1}$$

Other sources and sinks for financing, such as initial and variation margin collateral, could also be included, but we leave that as an exercise.

Fast forward to T_1 and the dealer balance sheet is given by:

$$A(q) = A + qX^+(q) - W_K(q)$$
$$L(q) = L + qX^-(q) + W_F(q)$$

with notation in table B. Due to the possibility of counterparty default, the actual derivative cashflow at T_1 is given by:

$$X(q) = X_c - 1_{B(q)}(1 - \beta(q))X_c^+ \tag{2}$$

which contributes to the assets if $X(q) \geqslant 0$, and to the liabilities otherwise. The counterparty default event $B(q)$ and recovery $\beta(q)$ can be taken as exogenous or derived from the counterparty balance sheet in the same way that:

$$D(q) = \{A(q) \leqslant L(q)\} \quad \text{and} \quad \rho(q) = A(q)/L(q)$$

are derived from the dealer balance sheet.

The T_1 assets change by $W_K(q)$ as a consequence of increasing or decreasing the T_0 equity by $U_K(q)$. Likewise, as a result of borrowing $U_F(q) \geqslant 0$ or repaying $U_F(q) \leqslant 0$ at the rate $r_F(q)$ agreed at T_0, the T_1 liabilities change by $W_F(q) = U_F(q)(1 + r_F(q))$. From now on, we treat $r_F(q)$ as exogenous and $W_F(q)$ as endogenous.

We have seen how the derivative and its financing create balance sheet feedback and join the dealer and counterparty balance sheets via their default events and recoveries. This allows a default of the counterparty to trigger dealer default and vice versa under certain circumstances. Consequently, the net cashflow $\mathrm{CF}(q)$ at T_1 between the dealer and the counterparty per unit of derivative is:

$$\mathrm{CF}(q) = X_c - 1_{B(q)}(1 - \beta(q))X_c^+ + 1_{D(q)}(1 - \rho(q))X_c^- \tag{3}$$

Figure 1 shows the resulting T_0 and T_1 cashflows between all the different model agents from a dealer perspective.

The cost of debt and equity

In this section, we make $S_F(q)$ and $W_K(q)$ endogenous. To do so requires a concept of fair valuation combined with the assumption that debt and equity are priced at their respective fair values.

A. A summary of the balance sheet notation used

Parameter	Description
Exogenous	
T_0, T_1	Period start and end times
A_0, L_0	T_0 legacy assets and liabilities
A, L	T_1 legacy assets and liabilities
q	Units of derivative $q \geqslant 0$
X_c	T_1 contractual derivative cashflow
$U(q)$	T_0 derivative price given q
$U_F(q)$	T_0 debt financing of $U(q)$
$U_S(q)$	T_0 off-balance sheet financing of $U(q)$
$U_K(q)$	T_0 equity financing of $U(q)$
$W_F(q)$	T_1 change in liabilities due to T_0 debt financing
$W_K(q)$	T_1 change in assets due to T_0 equity financing
Endogenous	
$D(q), \rho(q)$	T_1 dealer default event and recovery given q
$B(q), \beta(q)$	T_1 counterparty default event and recovery given q
$X(q)$	T_1 actual derivative cashflow given q
$A(q), L(q)$	T_1 assets and liabilities given q

Here, $X_c \geqslant 0$ is a payment to the dealer, $U(q) \geqslant 0$ represents a payment to the counterparty, and $U_f(q) \geqslant 0$ means the dealer is increasing the amount of financing for $f = S, F, K$. A negative sign reverses the payment direction

■ **Fair valuation.** A fair valuation rule is a linear and monotone mapping of a random T_1 cashflow amount X to a certain T_0 amount $\mathrm{FV}(X)$. It is well known that all such mappings are given by a discounted risk-neutral expectation:

$$\mathrm{FV}(X) = \delta\mathbb{E}^*[X] \tag{4}$$

where $\delta = \mathrm{FV}(1)$ is the fair value of a zero-coupon bond and $\mathbb{E}^*$ is the expectation with respect to a risk-neutral measure $P^* \sim P$. The period risk-free rate r is written as $\delta = 1/(1+r)$.

■ **The cost of debt.** Let $D^c(q)$ indicate dealer survival at T_1. If debt is priced at its fair value, then the debt-financing spread $S_F(q) = r_F(q) - r$ can be determined via the relation:

$$\delta\mathbb{E}^*[(1_{D^c(q)} + \rho(q)1_{D(q)})(1 + r + S_F(q))] = 1 \tag{5}$$

From the next section onwards, we will focus on the small trade limit $q \to 0$. In this case, we write $\nu_F = S_F(0)$, $D = D(0)$ and $\rho = \rho(0)$, and obtain:

$$\nu_F = \frac{\mathbb{E}^*[1_D(1-\rho)]}{\delta(1 - \mathbb{E}^*[1_D(1-\rho)])}$$

The corresponding debt discount factor is then defined as $\delta_F = 1/(1 + r + \nu_F)$.

■ **The cost of equity.** We start by defining the random variable Z via the relation $A - L = (A_0 - L_0)(1 + r + Z)$. This represents the excess return over the risk-free rate of the net portfolio of the legacy assets and liabilities. If A, L are priced at fair values, then $\mathbb{E}^*[Z] = 0$, as expected from a risk-neutral measure.

From a shareholder perspective, however, the legacy balance sheet performance is measured by the ROE. This is defined as:

$$\mathrm{ROE} = \frac{\mathrm{FV}((A-L)^+)}{A_0 - L_0} - 1 \tag{6}$$

To obtain a neat expression for ROE, we introduce a dealer survival measure P^S of the type used by Schönbucher (2003) via the relation:

$$\delta_S\mathbb{E}^S[X] = \delta\mathbb{E}^*[X1_{D^c}]$$

where $\delta_S = \delta P^*(D^c)$ is the fair value of a dealer survival-contingent zero-coupon bond. The single-period hazard rate ν_S can now be defined via the relation $\delta_S = 1/(1 + r + \nu_S)$ and then simplified to $\nu_S = P^*(D)/\delta P^*(D^c)$. Letting $\nu_K = \mathbb{E}^S[Z]$ denote the excess expected return of the portfolio of legacy assets and liabilities conditional on dealer survival allows us to rewrite (6) as:

$$\mathrm{ROE} = \delta_S(\nu_K - \nu_S)$$

The corresponding equity discount factor is written as $\delta_K = 1/(1 + r + \nu_K)$.

Finally, we let $U_K(q)$ have the same distribution as the legacy balance sheet such that $W_K(q) = U_K(q)(1 + r + Z)$.

■ **Next steps.** As it stands, the exogenous model parameters consist of the contractual payoff X_c, the quantity q, the price $U(q)$ and the dealer and counterparty legacy balance sheets A, L, along with their respective financing strategies. This is sufficient to calculate ROE and other balance sheet metrics as a function of q for a price-taking dealer who takes $U(q)$ as given. In the next sections, we will take an important step towards determining the price by calculating the firm and shareholder breakeven prices that leave their respective wealths unchanged.

The firm breakeven price

We fix all of the remaining exogenous model parameters listed in the previous subsection (except q) and calculate the total fair value of the cashflows in figure 1 going into and out of the firm at T_0 and T_1. By (3), the resulting firm wealth $F(q)$ at T_0 is given by:

$$F(q) = \delta\mathbb{E}^*[q(X_c - 1_{B(q)}(1-\beta(q))X_c^+ + 1_{D(q)}(1-\rho(q))X_c^-)] - U(q)$$

To obtain something more tractable, we focus on trades that are small compared with the size of the legacy balance sheet. More specifically, we let $u = \mathrm{d}U/\mathrm{d}q(0)$ and calculate the marginal firm wealth function $\mathrm{d}F/\mathrm{d}q(0) = f(u)$ with:

$$f(u) = \delta\mathbb{E}^*[X_c - 1_B(1-\beta)X_c^+ + 1_D(1-\rho)X_c^-] - u \tag{7}$$

Defining the marginal risk-free fair value $v_r^* := \delta\mathbb{E}^*[X_c]$ before solving $f(u) = 0$ allows us to state the following proposition.

PROPOSITION 1 *The marginal firm breakeven price $\hat{u}_{\mathrm{firm}}$ is given by:*

$$\hat{u}_{\mathrm{firm}} = v_r^* - \mathrm{cva} + \mathrm{dva}$$

where:

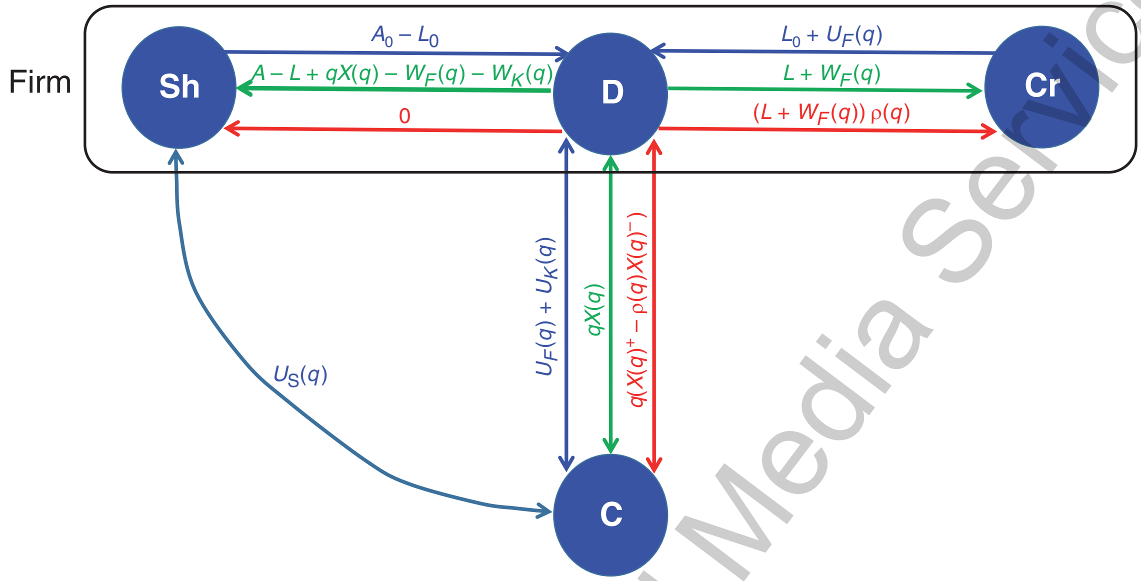
$$\mathrm{cva} = \delta\mathbb{E}^*[1_B(1-\beta)X_c^+]$$
$$\mathrm{dva} = \delta\mathbb{E}^*[1_D(1-\rho)X_c^-]$$

are the credit and debit valuation adjustments, respectively.

PROOF See Kjaer (2017). □

The single-period credit and debit valuation adjustment formulas in proposition 1 may seem simple, but they incorporate the full right- and wrong-way risk ultimately implied by the joint P^*-distribution of X_c as well as the dealer and counterparty legacy assets and liabilities. In the small trade limit, the firm breakeven price only depends on the legacy balance sheets via B, β, D and ρ, so we have taken the first step towards a reduced-form model. To complete this process, we have to make the joint distributions of these parameters and X_c exogenous to our model. In many practical implementations, the recoveries, default events and trade cashflow are taken to be independent of each other.

1 Cashflows between the dealer ('D') and its shareholders ('Sh'), creditors ('Cr') and counterparty ('C') at T_0 (blue), T_1 dealer survival (green) and T_1 dealer default (red)



The firm is highlighted by a black frame. Cashflows that can go in either direction are represented by bidirectional arrows

The shareholder breakeven price

By following similar steps as in the previous section, we can calculate the marginal shareholder wealth $g(u_F, u_S, u_K)$ at T_0, where:

$$u_F = \frac{\mathrm{d}U_F}{\mathrm{d}q}(0), \qquad u_S = \frac{\mathrm{d}U_S}{\mathrm{d}q}(0), \qquad u_K = \frac{\mathrm{d}U_K}{\mathrm{d}q}(0)$$

satisfy the marginal financing constraint $u_S + u_F + u_K = u$. Kjaer (2017) extends the proofs of Andersen *et al* (2019) to show that:

$$g(u_F, u_S, u_K) = \delta_S \mathbb{E}^S[X - u_F/\delta_F - u_S/\delta_S - u_K/\delta_K] \tag{8}$$

To simplify our later analysis, we now express the financing strategy in terms of financing weights $u_F = \alpha_F u$, $u_S = \alpha_S u$ and $u_K = \alpha_K u$ and rewrite the wealth as:

$$g_\alpha(u) = \delta_S \mathbb{E}^S[X - u/\delta_\alpha] \tag{9}$$

where $\delta_\alpha = 1/(1 + r + \nu_\alpha)$ is a weighted average cost of capital (WACC) discount factor computed from the WACC spread: $\nu_\alpha = \alpha_S \nu_S + \alpha_F \nu_F + \alpha_K \nu_K$.

The shareholders should only accept the new derivative if $g_\alpha(u) \geqslant 0$, which yields the following proposition.

Proposition 2 *The marginal shareholder breakeven price $\hat{u}_{\mathrm{sh}}$ is given by:*

$$\begin{aligned}\hat{u}_{\mathrm{sh}} &= \delta_\alpha \mathbb{E}^S[X] \\ &= v_r^S - \mathrm{fva}_\alpha - \mathrm{fcva}_\alpha\end{aligned}$$

where:

$$\begin{aligned} v_r^S &= \delta \mathbb{E}^S[X_{\mathrm{c}}] \\ \mathrm{fva}_\alpha &= \delta_\alpha \nu_\alpha v_r^S \\ \mathrm{fcva}_\alpha &= \delta_\alpha \mathbb{E}^S[1_B(1-\beta)X_{\mathrm{c}}^+]\end{aligned}$$

are the shareholder risk-free price, the financing valuation adjustment and the financing discounted credit valuation adjustment, respectively.

Proof See Kjaer (2017). □

Proposition 2 is interesting in two ways. First, it uses a shareholder risk-free price v_r^S, which is distinct from v_r^*. We will discuss this further in the next section. Second, $\hat{u}_{\mathrm{sh}}$ contains a total financing valuation adjustment determined by the dealer WACC.

For more details on the deeper connections between firm and shareholder breakeven prices, see Kjaer (2017).

A financing strategy with capital valuation adjustment

In this section, we will consider a specific financing strategy that involves the marginal regulatory capital requirement $k = \mathrm{d}K(q)/\mathrm{d}q(0)$ and the resulting capital valuation adjustment explicitly. More specifically, we set:

$$\begin{aligned} u_F &= \delta_F \mathbb{E}^S[X] - k \\ u_f &= -\delta_f(\nu_K - \nu_F)k \\ u_K &= k \end{aligned} \tag{10}$$

where, again, $f \in \{F, K, S\}$. We now obtain the following proposition.

Proposition 3 *The marginal shareholder breakeven price $\hat{u}_{\mathrm{sh}}$ for the financing strategy (10) is given by:*

$$\hat{u}_{\mathrm{sh}} = v_r^S - \mathrm{fva} - \mathrm{fcva} - \mathrm{kva}$$

where:

$$\begin{aligned} \mathrm{fva} &= \delta_F \nu_F v_r^S \\ \mathrm{fcva} &= \delta_F \mathbb{E}^S[1_B(1-\beta)X_{\mathrm{c}}^+] \\ \mathrm{kva} &= \delta_f(\nu_K - \nu_F)k \end{aligned}$$

are the funding, funding discounted credit and capital valuation adjustments, respectively.

Proof See Kjaer (2017). □

These formulas are single-period versions of the continuous-time valuation adjustments derived in Burgard & Kjaer (2013) and Green *et al* (2014). While simple, they incorporate right- and wrong-way risk by using the joint distribution of the counterparty default and the derivative payoff, conditional on dealer survival. We now look into these adjustments in more detail.

■ **The shareholder risk-free price.** This price differs from the risk-free fair valuation v_r^* in that it is computed under the dealer survival measure P^S rather than P^*. Kjaer (2018) provides an example in which the two measures are different due to a jump in a spot asset price at dealer default.

■ **The funding valuation adjustment.** This is a specialisation of the general financing valuation adjustment from the previous section. It is evaluated under the survival measure and can be positive or negative depending on whether the deal increases or reduces dealer financing requirements.

■ **The financing discounted credit valuation adjustment.** The financing discounted credit valuation adjustment is always non-negative and is similar to the firm credit valuation adjustment; however, it is evaluated under P^S rather than P^* and is discounted with δ_F rather than δ.

■ **The capital valuation adjustment.** The capital valuation adjustment, kva, is always non-negative. Its accrual rate $\nu_K - \nu_F$ consists of two components. The rate ν_K is the excess return of the balance sheet assets liquidated at T_0 to free the regulatory capital and thus represents an opportunity cost. The rate ν_F is due to the reduction in debt financing by using equity instead. The choice of financing method f for the kva itself is tied to how it is managed. More specifically, $f = S$ means the kva is released immediately at T_0. Choosing $f = F$ means it is held on the balance sheet and used in lieu of debt. Finally, $f = K$ corresponds to reserving it in retained earnings and letting it count towards the regulatory capital requirement. As a consequence, fewer legacy assets need to be liquidated and the return on these assets is awarded to kva; this is how it comes to grow at the rate $1/\delta_K$. Such a case was previously derived by Albanese *et al* (2016).

In the special case when $f = F$, we can incorporate the financing relief due to equity financing into the funding valuation adjustment such that:

$$\hat{u}_{\text{sh}} = v_r^S - \delta_F \nu_F (v_r^S - k) - \delta_F \mathbb{E}^S[1_B(1-\beta)X_c^+] - \delta_F \nu_K k$$

Static and dynamic xva management

The financing strategies presented so far constitute static management in that the amount $\hat{u}_{\text{sh}}/\delta_\alpha = \mathbb{E}^S[X]$ released at T_1 is determined at T_0. At the same time, the actual trade cashflow X is also released, resulting in a volatile net release of $X - \mathbb{E}^S[X]$.

To remove this volatility, we employ dynamic management by hedging the payoff X at the marginal cost u_h. This cost has the opposite sign to u and is financed with debt, since the debt account is part of the replication strategy. If the derivative is also financed with debt, then it is straightforward to show $\hat{u}_{\text{sh}} = u_h$. For the general case, it is shown in Kjaer (2017) that:

$$\hat{u}_{\text{sh}} = \delta_\alpha u_h / \delta_F \tag{11}$$

We can take u_h as an exogenous cost of a cleared back-to-back trade also with payoff X, which could then include any clearing house initial margin. Alternatively, we can replicate the payoff with simpler traded instruments. By Kjaer (2017) this yields $u_h = \delta_F \mathbb{E}^Q[X]$, where $Q \sim P^S$ is a survival measure implied by the replication strategy. Inserting this u_h into (11) yields:

$$\hat{u}_{\text{sh}} = \delta_\alpha \mathbb{E}^Q[X] \tag{12}$$

B. The single-period and continuous-time notation

Single period	Continuous time
$(\Omega, \mathcal{F}, P)$	$(\Omega, \mathcal{F}, \mathcal{F}_t, P)$ for $0 \leqslant t \leqslant T$
r	$r(t)$
δ	$D_r(t,T) = \exp(-\int_t^T r(u)\,\mathrm{d}u)$
X_c	$x_c(t)$ (cashflow stream)
$v_r^* = \delta\mathbb{E}^*[X_c]$	$V_r^*(t) = \mathbb{E}^*[\int_t^T D_r(t,u)x_c(u)\,\mathrm{d}u \mid \mathcal{F}_t]$
D, B	$\{\tau_D \leqslant t\}, \{\tau_C \leqslant t\}$
$\delta_S\mathbb{E}^S[X_c] = \delta\mathbb{E}^*[X_c 1_{D^c}]$	$\mathbb{E}_t^S[D_S(0,t)x_c(t)] = \mathbb{E}^*[D_r(0,t)x_c(t)1_{\{\tau_D > t\}}]$
$\nu_f,\ f = S, F, K$	$\nu_f(t),\ f = S, F, K$
$\delta_f = 1/(1 + r + \nu_f)$	$D_f(t,T) = \exp(-\int_t^T (r(u) + \nu_f(t))\,\mathrm{d}u)$
$\mathrm{ROE} = \delta_S(\nu_K - \nu_S)$	$\mathrm{roe}(t) = \nu_K(t) - \nu_S(t)$
k	$K(t)$

so the shareholder breakeven price formula for a hedged derivative is still given by propositions 2 and 3, with $P^S = Q$.

Continuous time

Here, we state the continuous-time versions of propositions 1 and 3 from Kjaer (2018).

■ **Notation.** We start by listing our notation in table B. The valuation adjustments are denoted by uppercase acronyms to distinguish them from their single-period counterparts. All calculations take place at $t = 0$ unless stated otherwise.

■ **Firm breakeven price.** At dealer and counterparty default times, τ_D and τ_C, there are final closeout cashflows of $G_D(\tau_D)$ and $G_C(\tau_C)$, which depend on their recoveries $\rho(\tau_D)$ and $\beta(\tau_C)$. With this assumption in place, the continuous-time extension of proposition 1 reads as follows.

Proposition 4 *The firm breakeven price $\hat{U}_{\text{firm}}$ can be written as:*

$$\hat{U}_{\text{firm}} = V_r^* - \text{CVA} + \text{DVA}$$

with:

$$\text{CVA} = \mathbb{E}^*[D_r(0,\tau_C)(V_r^*(\tau_C) - G_C(\tau_C))1_{\{\tau_C < \tau_D, \tau_C \leqslant T\}}]$$
$$\text{DVA} = \mathbb{E}^*[D_r(0,\tau_D)(V_r^*(\tau_D) - G_D(\tau_D))1_{\{\tau_D < \tau_C, \tau_D \leqslant T\}}]$$

Proof See Kjaer (2018). □

■ **Shareholder breakeven price.** The continuous-time shareholder breakeven price is expressed in terms of expectations $\mathbb{E}_t^f$ with respect to a financing measure $\mathrm{d}P_t^f = Z_f(t)\,\mathrm{d}P_t^S$ for $f = S, F, K$ with the Radon-Nikodyn derivative:

$$Z_f(t) = \frac{D_S(0,t)}{D_f(0,t)} \bigg/ \mathbb{E}_t^S\left[\frac{D_S(0,t)}{D_f(0,t)}\right]$$

To state the continuous-time equivalent of proposition 3, we need the shareholder risk-free price:

$$V_r^F(t) := \int_t^T \mathbb{E}_u^F[D_r(t,u)x_c(u) \mid \mathcal{F}_t]\,\mathrm{d}u$$

With some effort, we can now prove the following proposition.

Proposition 5 *The shareholder breakeven price $\hat{U}_{\text{sh}}$ is given by:*

$$\hat{U}_{\text{sh}} = V_r^F - \text{FVA} - \text{FCVA} - \text{KVA}$$

with:

$$\text{FVA} = \int_0^T \mathbb{E}_t^F[\nu_F(t)D_F(0,t)V_r^F(t)1_{\{\tau_C>t\}}]\,dt$$
$$\text{FCVA} = \mathbb{E}_{\tau_C}^F[D_F(0,\tau_C)(V_r^F(\tau_C) - G_C(\tau_C))1_{\{\tau_C\leqslant T\}}]$$
$$\text{KVA} = \int_0^T \mathbb{E}_t^f[(\nu_K(t) - \nu_F(t))D_f(0,t)K(t)1_{\{\tau_C>t\}}]\,dt$$

Proof See Kjaer (2018). □

These valuation adjustments are model independent and incorporate right- and wrong-way risk. Setting $f = S$ and identifying $\nu_S(t)$ with a deterministic dealer hazard rate replicates the KVA formula of Green *et al* (2014), albeit derived from rather different assumptions. These authors use a deterministic hazard rate model with independent defaults, which implies that all dealer financing measures are equal to the risk-neutral measure due to no-way risk.

Static and dynamic XVA management. The continuous-time financing strategies used by Kjaer (2018) essentially release the XVA over time by paying a cashflow stream equal to the XVA formula integrands in proposition 5, grown at their cost of financing (unless $f = S$, in which case the entire KVA is paid at $t = 0$). Since these integrands are expectations computed at $t = 0$, this is a static management method. In particular, if stochastic moves in market factors increase or decrease $K(t)$ by more than its expected value at $t = 0$, then the realised ROE will be below or above the target.

The continuous-time equivalent of the single-period hedging objective discussed in the previous section entails setting up a self-financing strategy that generates the cashflow stream $x_c(t) + (\nu_K(t) - \nu_F(t))K(t)$ while both parties are alive, and $G_C(\tau_C)$ at counterparty default. The semi-replication itself is debt financed and assumes all randomness of $K(t)$ comes from movements in tradable instruments. This approach thus guarantees the instantaneous realised ROE always equals the target. As in the single-period model, the breakeven price when hedging is given by using the measure induced by the replication in the formulas of proposition 5 while leaving the discounting intact. If pure debt financing is used, then this price equals the cost of setting up the hedge. This justifies the approach taken by Burgard & Kjaer (2013), who identify the 'shareholder economic value' with the cost of setting up a replicating portfolio. Kjaer (2018) extends the semi-replication methodology to a model for which a spot asset price jumps at both dealer and counterparty default. The resulting hedging cost excludes the jump at dealer default, as expected from a dealer survival measure.

Dealer level KVA. So far, the regulatory capital requirement $K(t)$ has been computed at the counterparty level, and the aggregation across counterparties is assumed to have been computed by a simple summation. The Basel III BA-CVA capital requirements defined in Basel Committee on Banking Supervision (2017) have dealer-level capital $K_D(t)$ being calculated as a non-linear aggregation over the counterparty level capital requirements $K_C(t)$ for N_C counterparties. At a future time t, this aggregate requirement will include only those counterparties that have not yet defaulted. It follows that $K_D(t) = K_D(t, \xi(t))$, where $\xi(t)$ is a binary vector of N_C elements $\xi_C(t) = 1_{\{\tau_C>t\}}$. The dealer-level KVA_D then becomes:

$$\text{KVA}_D = -\int_0^T \mathbb{E}_t^f[(\nu_K(t) - \nu_F(t))D_f(0,t)K_D(t,\xi(t))]\,dt \quad (13)$$

The formula (13) has many structural similarities to the dealer-level funding valuation adjustment studied by Burgard & Kjaer (2017), including the exponential computational complexity in N_C. Thus, some kind of approximation would be needed to calculate (13) in practice.

C. Counterparty (CP) parameters used

CP rating	λ_C	Recovery	SA-CCR RW	BA-CVA RW
AA	100bp	40%	20%	3%
BB	250bp	40%	100%	7%

D. Shareholder valuation adjustments for the ITM test swap

XVA	AA CP Unmargined	AA CP Margined	BB CP Unmargined	BB CP Margined
FCVA	−1.91	−0.12	−4.51	−0.27
FVA	−1.36	0.00	−1.29	0.00
SA-CCR KVA ($f = F$)	−1.08	−0.13	−5.12	−0.60
SA-CCR KVA ($f = S$)	−1.07	−0.13	−5.04	−0.60
SA-CCR KVA ($f = K$)	−0.77	−0.09	−3.66	−0.45
BA-CVA KVA ($f = F$)	−3.83	−0.47	−8.55	−1.06
BA-CVA KVA ($f = S$)	−3.78	−0.47	−8.46	−1.05
BA-CVA KVA ($f = K$)	−2.93	−0.37	−6.58	−0.83

The results are given in bp of notional per year

E. Shareholder valuation adjustments for the ATM test swap

XVA	AA CP Unmargined	AA CP Margined	BB CP Unmargined	BB CP Margined
FCVA	−1.11	−0.12	−2.60	−0.29
FVA	−0.06	0.00	−0.06	0.00
SA-CCR KVA ($f = F$)	−0.74	−0.13	−3.46	−0.60
SA-CCR KVA ($f = S$)	−0.72	−0.13	−3.41	−0.60
SA-CCR KVA ($f = K$)	−0.51	−0.09	−2.42	−0.45
BA-CVA KVA ($f = F$)	−2.51	−0.47	−5.60	−1.06
BA-CVA KVA ($f = S$)	−2.48	−0.47	−5.53	−1.05
BA-CVA KVA ($f = K$)	−1.88	−0.37	−4.22	−0.83

The results are given in bp of notional per year

Results

Tables D–F show how shareholder XVA in general and SA-CCR and BA-CVA KVA in particular can affect pricing significantly. Each table in turn prices out-of-the-money (OTM), at-the-money (ATM) and in-the-money (ITM) 10-year US$100 million notional vanilla test swaps, with the dealer paying fixed semi-annually and receiving US$ three-month Libor quarterly. The ATM fixed rate is 2.3395%, with the ITM and OTM strikes set at 40 basis points either side. For completeness, we calculate KVA for all three financing methods $f = F, S, K$.

For pricing, we use a standard reduced-form model, where interest rates follow a one-factor Hull-White model with a deterministic additive basis between the Libor and overnight index swap (OIS) short rates as well as independent dealer and counterparty default times driven by constant hazard rates λ_D and λ_C. The dealer hazard rate is $\lambda_D = 100$bp, its funding spread is $\nu_F = 60$bp and its ROE target is 10% per annum. Two counterparties are considered, with the parameters given in table C. Finally, we consider both the unmargined and the margined cases. For the latter, we use zero thresholds, minimum transfer amounts and independent amounts combined with a 10-business-day margin period of risk.

The resulting SA-CCR exposures at default are used in the Basel III BA-CVA capital formula defined by Basel Committee on Banking Supervision (2017). To emulate the diversification benefit of having a book with a large number of similar counterparties, we use the approximation $K(t) = \rho\,\text{SCVA}(t)$, where $\rho = 0.5$ and $\text{SCVA}(t)$ is the counterparty level BA-CVA capital requirement.

F. Shareholder valuation adjustments for the OTM test swap

	AA CP		BB CP	
XVA	Unmargined	Margined	Unmargined	Margined
FCVA	−0.62	−0.13	−1.43	−0.31
FVA	1.23	0.00	1.18	0.00
SA-CCR KVA ($f = F$)	−0.48	−0.13	−0.48	−0.13
SA-CCR KVA ($f = S$)	−0.47	−0.13	−0.47	−0.13
SA-CCR KVA ($f = K$)	−0.33	−0.09	−0.33	−0.09
BA-CVA KVA ($f = F$)	−1.59	−0.47	−3.54	−1.06
BA-CVA KVA ($f = S$)	−1.57	−0.47	−3.49	−1.05
BA-CVA KVA ($f = K$)	−1.17	−0.37	−2.61	−0.83

The results are given in bp of notional per year

As feared, the cost of financing the regulatory capital becomes very high if either the counterparty is risky or the trade is unmargined. The worst case is the ITM swap in table D, where the total XVA adjustment is 19.47bp of notional annually (13.67bp of which is KVA) when trading uncollateralised with the BB counterparty while using the KVA in lieu of debt financing ($f = F$). This cost can be reduced in two ways. First, if the two parties can sign a perfect two-way credit support annex for variation margin, then the total XVA in the worst-case example is reduced by a factor of 10 to around 2bp of notional annually. Second, using the KVA itself in lieu of regular capital allows the dealer to set $f = K$, which reduces KVA f by roughly one-third in the examples above.

Finally, we note SA-CCR KVA is 30–60% of the BA-CVA KVA, which is consistent with the financial crisis folklore that two-thirds of counterparty credit risk losses were due to CVA risk rather than outright defaults.

Conclusion

We have developed a dealer balance sheet model and used it to derive consistent and model-independent marginal firm and shareholder breakeven prices and valuation adjustments for a new derivative contract. The former is what an external party should pay to acquire the derivative and its financing. The latter ensures shareholders are not worse off when they account for the cost of financing the derivative, including regulatory capital. The resulting XVAs are valid regardless of whether the XVA terms are managed statically or dynamically. Importantly, the KVA can be discounted at a comparably high ROE, provided it is reserved on the balance sheet. Even so, our examples show it can be of considerable magnitude compared with other shareholder XVA terms. As a major driver of the derivative profitability, we agree with Green *et al* (2014) that KVA should be calculated and managed using one of the static or dynamic methods presented in this article. ■

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