What do central counterparties default funds really cover? A network-based stress test answer

APPENDIX: MERTON-LIKE MODEL FOR DAILY BALANCE SHEETS

In order to compute the financial position of each CM at each date t, we have to obtain daily "dynamic" values for both total assets and liabilities starting from the information disclosed periodically. To this end, we use a Merton-like model that estimates the value of a firm's equity according to Black and Scholes option pricing theory. The main insight of this approach is that the equity of a firm can be modeled as the price of a call option on the assets of the firm, with a strike price equal to the notional amount of debt issued by the company (Merton, 1974). Indeed, shareholders are the residual owners of a company: the value of the assets above the debt will be paid out to them, otherwise they will get nothing.

Here we resort to a variation of the original Merton model where we remove the assumption that default (or insolvency) can only occur at the maturity of the debt. We suppose instead that default occurs the first time the firm's total assets fall below the default point, *i.e.*, the notional value of debt. As suggested by Bharath and Shumway (2008), we approximate the face value of the firm's debt with the book value of the firm's total liabilities. Pricing techniques for barrier options, whose payoff depends on whether the underlying asset price reaches a certain level during a specified time interval, can be used for our purpose. In particular, we consider down-and-out call options, *i.e.*, knock-out call options that cease to exist if the asset price decreases falls below the barrier. Following (Tudela and Young, 2005), we set the barrier equal to the firm's total liabilities and the maturity to $T = 1$ year.

We build on the assumption that the total assets of a generic firm i follow a geometric Brownian motion

$$dA_i(t) = \mu_i^{(A)} A_i(t)dt + \sigma_i^{(A)} A_i(t)dW, \tag{1}$$

where $\mu_i^{(A)}$ is the expected continuously compounded return on A_i, $\sigma_i^{(A)}$ is the volatility of A_i and dW is a standard Wiener process. According to the Black and Scholes pricing model, the price of the considered down-and-out call option is given by:

$$E_i(t) = N[d_+]A_i(t) - N[d_-]L_i e^{-rT} - N[y]A_i(t)\left(\frac{L_i}{A_i(t)}\right)^{2\lambda} + N[\tilde{y}]L_i e^{-rT}\left(\frac{L_i}{A_i(t)}\right)^{2\lambda-2}, \tag{2}$$

where

$$d_\pm[A_i(t), \sigma_i^{(A)}] = \frac{1}{\sigma_i^{(A)}\sqrt{T}}\left[\ln\left(\frac{A_i(t)}{L_i}\right) + \left(r \pm \frac{1}{2}(\sigma_i^{(A)})^2\right)T\right], \qquad \lambda[\sigma_i^{(A)}] = \frac{r}{(\sigma_i^{(A)})^2} + \frac{1}{2},$$

$$y[A_i(t), \sigma_i^{(A)}] = \frac{1}{\sigma_i^{(A)}\sqrt{T}}\ln\left(\frac{L_i}{A_i(t)}\right) + \lambda\sigma_i^{(A)}\sqrt{T}, \qquad \tilde{y}[A_i(t), \sigma_i^{(A)}] = y - \sigma_i^{(A)}\sqrt{T},$$

N indicates the cumulative function of the standard normal distribution and r is the risk-free rate. Moreover, it can be shown that the following relation holds between the equity volatility $\sigma_i^{(E)}$ and the assets volatility $\sigma_i^{(A)}$ (Jones et al., 1984):

$$\begin{aligned} E_i(t)\sigma_i^{(E)} &= A_i(t)\sigma_i^{(A)}\frac{\partial E_i(t)}{\partial A_i(t)} = \\ &= N[d_+]A_i(t)\sigma_i^{(A)} + N[y]\left[(2\lambda-1)A_i(t)\sigma_i^{(A)}\left(\frac{L_i}{A_i(t)}\right)^{2\lambda}\right] + N[\tilde{y}]\left[(2-2\lambda)A_i(t)\sigma_i^{(A)}e^{-rT}\left(\frac{L_i}{A_i(t)}\right)^{2\lambda-1}\right]. \end{aligned} \tag{3}$$

In the model we adopt, the option value $E_i(t)$ is observed on the market as the total current value of the firm's equity, while its volatility $\sigma_i^{(E)}$ can be easily estimated. On the other hand, the unknown variables are the current value of assets $A_i(t)$ and its volatility $\sigma_i^{(A)}$. They can be estimated by inverting the two nonlinear equations (2) and (3).

In the implementation of the Merton model, the following input parameters have been used: $E_i(t)$ has been approximated as the firm's market capitalization (if the firm is a listed company) or equity (as reported in the last available balance sheet); $\sigma_i^{(E)}$ has been proxied as the volatility of the market capitalization (if the company is listed), otherwise as the volatility of a reference index [1], as suggested by Bharath and Shumway (2008); L_i has been represented as the total liabilities of the firm as reported in the last available balance sheet; $\mu_i^{(A)}$ has been estimated as the annual return on assets of the company.

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[1] If the CM is Italian we use the FTSE Italia All Share Banks Index, otherwise we take the EURO STOXX Banks Eur Index.